

# Notes for marking Level J

In the expansion and factorisation exercises J1-60, the exact form of the answers can vary depending on the order of letters, brackets and signs. Only a typical form of each answer is shown here, and equivalent answers should be marked correct (provided they are fully factorised).

## EQUIVALENT ANSWERS

Examples	Notes
$b - a$ or $-a + b$ $(3 + x)(2 + x)$ or $(x + 3)(x + 2)$ $3 - x - x^2$ or $-x^2 - x + 3$	Terms may be added in a different order. Note: it is normally recommended to write answers in alphabetic order. Answers with powers are normally written in descending order of powers.
$(2x + 4)(3x + 1)$ or $(3x + 1)(2x + 4)$ $(x - 1)(x + 1)$ or $(x + 1)(x - 1)$	Terms may be multiplied in a different order.
$\frac{1}{3}x - \frac{2}{5}y$ or $\frac{x}{3} - \frac{2y}{5}$ $\frac{3x + 1}{3}$ or $x + \frac{1}{3}$	Fractions may be written either as coefficients or as algebraic fractions.
$(4 - x)(1 + x)$ or $-(x - 4)(x + 1)$	Negative signs can appear inside or outside brackets.

## FORMAL DEFINITION

Two answers are equivalent if they give the same result no matter which values are substituted:

- $(x + 2)(x + 3)$  and  $(x + 3)(x + 2)$  **are** equivalent, e.g. substituting  $x = 10$  gives

$$\left. \begin{array}{l} (x + 2)(x + 3) = 12 \times 13 = 156 \\ (x + 3)(x + 2) = 13 \times 12 = 156 \end{array} \right\} \text{agree (also agree for other values of } x \text{)}$$

- $(x + 2)^2$  and  $x^2 + 4$  are **not** equivalent, e.g. substituting  $x = 10$  gives

$$\left. \begin{array}{l} (x + 2)^2 = (12)^2 = 144 \\ x^2 + 4 = 10^2 + 4 = 104 \end{array} \right\} \text{disagree (also disagree for most other values of } x \text{)}$$

J 1-4

1	2
<p>(1) <math>a^{12}</math></p> <p>(2) <math>-8a^6b^3c^9</math></p> <p>(3) <math>15a^5b^5</math></p> <p>(4) <math>-a^3b^5</math></p>	<p>(1) <math>6xy^2z+12x^2yz</math></p> <p>(2) <math>10xyz^2-15xy^2z</math></p> <p>(3) <math>-12x^2y^2z^2+4x^3z^3</math></p> <p>(4) <math>-2a^2b^2c-2ab^2c^2-2a^2bc^2</math></p>
1	2
<p>(5) <math>-\frac{1}{3}a^3b^5</math></p> <p>(6) <math>a^2</math></p> <p>(7) <math>1</math></p> <p>(8) <math>\frac{4a^2}{3b}</math></p> <p>(9) <math>-\frac{8a}{b^2}</math></p> <p>(10) <math>\frac{5}{2y^2}</math></p>	<p>(5) <math>6x^2y^3z-5xy^2z^3</math></p> <p>(6) <math>4a^2b^4-\frac{3b^2}{a}</math></p> <p>(7) <math>3x-\frac{4}{x}</math></p> <p>(8) <math>3xy-\frac{4}{3}</math></p> <p>(9) <math>\frac{a^4}{2x}+ab^2-\frac{3ac}{2}</math></p>

3	4
<p>(1) <math>a^2 - b^2</math></p> <p>(2) <math>x^2 - 4y^2</math></p> <p>(3) <math>x^2 + 4x + 4</math></p> <p>(4) <math>18a^6 + 48a^3 + 32</math></p> <p>(5) <math>-2a^2 + 12ab - 18b^2</math></p>	<p>1.</p> <p>(1) <math>4a^2 + 9b^2 + c^2 - 12ab + 4ac - 6bc</math></p> <p>(2) <math>9x^2 + 4y^2 + z^2 - 12xy - 6xz + 4yz</math></p> <p>(3) <math>x^4 + 2x^3 - 5x^2 - 6x + 9</math></p>
3	4
<p>(6) <math>x^2 - \frac{1}{6}x - \frac{1}{6}</math></p> <p>(7) <math>3x^2 + 3x - 18</math></p> <p>(8) <math>-30x^2 - 25x + 20</math></p> <p>(9) <math>-b^2</math></p> <p>(10) <math>21a^2 - 19</math></p>	<p>2.</p> <p>(1) <math>4a^2 + 9b^2 + c^2 - 12ab + 6bc - 4ca</math></p> <p>(2) <math>4x^2 + 9y^2 + 16z^2 - 12xy - 24yz + 16zx</math></p> <p>(3) <math>4x^2 + 9y^2 + 25z^2 + 12xy + 30yz + 20zx</math></p> <p>(4) <math>x^4 - 4x^3 - 2x^2 + 12x + 9</math></p> <p>(5) <math>4x^4 - 12x^3 + 25x^2 - 24x + 16</math></p>

J 5-8

5	6
<p>(1) <math>a^2 + 4ab + 4b^2 - 9c^2</math></p> <p>(2) <math>a^2 - 6ab + 9b^2 - 25c^2</math></p> <p>(3) <math>a^2 - 8ab + 16b^2 - 4c^2</math></p>	<p>(1) <math>x^2 + 4xy + 4y^2 + 7ax + 14ay + 12a^2</math></p> <p>(2) <math>4a^2 - 4ac + c^2 - 4ab + 2bc - 8b^2</math></p>
5	6
<p>(4) <math>(a - 2c)(a - 2c)</math> <math>(a - 2c)(4b)</math> <math>a^2 - 4ac + 4c^2 - 16b^2</math></p> <p>(5) <math>a^2 + 10ac + 25c^2 - 9b^2</math></p> <p>(6) <math>(b - c)(b - c)</math> <math>(b - c)</math> <math>a^2 - b^2 + 2bc - c^2</math></p> <p>(7) <math>(c + d)(c + d)</math> <math>a^2 + 2ab + b^2 - c^2 - 2cd - d^2</math></p> <p>(8) <math>a^2 + 2ac + c^2 - b^2 + 2bd - d^2</math></p>	<p>(3) <div>19</div> <math>15a^2 + 30ab + 15b^2 + 19a + 19b - 10</math></p> <p>(4) <math>6a^2 - 12ab + 6b^2 - ac + bc - 15c^2</math></p> <p>(5) <div>6</div><div>4</div><div>6</div><div>4</div> <math>x^4 + 10x^3 + 35x^2 + 50x + 24</math></p> <p>(6) <math>x^4 + 12x^3 + 49x^2 + 78x + 40</math></p>

7	8
<p>(1) <math>a^3 + 3a^2 + 3a + 1</math></p> <p>(2) <math>a^3 + 6a^2 + 12a + 8</math></p> <p>(3) <math>a^3 + 15a^2 + 75a + 125</math></p> <p>(4) <math>8x^3 + 12x^2 + 6x + 1</math></p> <p>(5) <math>x^3 + 6x^2y + 12xy^2 + 8y^3</math></p>	<p>(1) <math>a^3 + 12a^2 + 48a + 64</math></p> <p>(2) <math>8a^3 - 12a^2b + 6ab^2 - b^3</math></p> <p>(3) <math>27a^3 + 54a^2b + 36ab^2 + 8b^3</math></p> <p>(4) <math>x^6 + 3x^4 + 3x^2 + 1</math></p> <p>(5) <math>8x^6 - 36x^4 + 54x^2 - 27</math></p>
7	8
<p>(6) <math>a^3 - 3a^2 + 3a - 1</math></p> <p>(7) <math>a^3 - 9a^2 + 27a - 27</math></p> <p>(8) <math>8a^3 - 12a^2 + 6a - 1</math></p> <p>(9) <math>8x^3 - 36x^2 + 54x - 27</math></p> <p>(10) <math>x^3 - 9x^2y + 27xy^2 - 27y^3</math></p>	<p>(6) <math>-a^3 - 3a^2 - 3a - 1</math></p> <p>(7) <math>-8a^3 - 12a^2b - 6ab^2 - b^3</math></p> <p>(8) <math>-x^6 + 9x^4 - 27x^2 + 27</math></p> <p>(9) <math>a^3 + a^2 + \frac{1}{3}a + \frac{1}{27}</math></p> <p>(10) <math>-8a^3 - 4a^2 - \frac{2}{3}a - \frac{1}{27}</math></p>

J 9–12

9	10
<p>(1) <math>x^6 - 12x^4 + 48x^2 - 64</math></p> <p>(2) <math>a^6 - 27a^4 + 243a^2 - 729</math></p> <p>(3) <math>x^6 - 12x^4y^2 + 48x^2y^4 - 64y^6</math></p>	<p>(1) <math>27a^3 + 9a^2 + a + \frac{1}{27}</math></p> <p>(2) <math>x^3 - 6x + \frac{12}{x} - \frac{8}{x^3}</math></p> <p>(3) <math>x^6 + 3x^3 + 3 + \frac{1}{x^3}</math></p> <p>(4) <math>64x^6 - 48x^4 + 12x^2 - 1</math></p>
9	10
<p>(4) <math>x^3 + 9x^2 + 23x + 15</math></p> <p>(5) <math>x^3 - 6x^2 + 11x - 6</math></p> <p>(6) <math>x^3 + 3x^2 - 10x - 24</math></p> <p>(7) <math>x^3 + 7x^2 + 14x + 8</math></p>	<p>(5) <math>4a^2 + 9b^2 + c^2 - 12ab - 6bc + 4ca</math></p> <p>(6) <math>4x^4 - 12x^3 - 7x^2 + 24x + 16</math></p> <p>(7) <math>4x^4 - 9x^2 + 24x - 16</math></p> <p>(8) <math>4x^4 - x^2 + 6x - 9</math></p>

11	12
<p>(1) <math>(x+5)(x+3)</math></p> <p>(2) <math>(x+7)(x+3)</math></p> <p>(3) <math>(a-5)(a-2)</math></p> <p>(4) <math>(a+5b)(a-2b)</math></p> <p>(5) <math>(x^2+2x-8)</math> <math>12(x+4)(x-2)</math></p> <p>(6) <math>a(x-4)(x+2)</math></p>	<p>(1) <math>(x+y-3)(x+y-2)</math></p> <p>(2) <math>(x+y-5)(x+y+3)</math></p> <p>(3) <math>(x+y+6)(x+y-3)</math></p> <p>(4) <math>(x+y-5)(x+y-2)</math></p> <p>(5) <math>(x-3)(x-2)</math></p>
11	12
<p>(7) <math>(x+y+5)(x+y+3)</math></p> <p>(8) <math>(x+y-4)(x+y+3)</math></p> <p>(9) <math>(2x+y+5a)(2x+y+2a)</math></p> <p>(10) <math>(x-10)(x+3)</math></p> <p>(11) <math>(x-1)(x-8)</math></p>	<p>(6) <math>(x-y+3z)(x-y+z)</math></p> <p>(7) <math>(x+y-5z)(x+y+3z)</math></p> <p>(8) <math>(a+b+5c)(a+b+c)</math></p> <p>(9) <math>(a+b-5c)(a+b-2c)</math></p> <p>(10) <math>(a-2b-5c)(a-2b-4c)</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

J 13–16

13	14
<p>(1) <math>(2x + 5)(x + 1)</math></p> <p>(2) <math>(6x - 1)(x + 5)</math></p> <p>(3) <math>(5x + 4)(2x - 3)</math></p> <p>(4) <math>(3x - 1)(2x - 5)</math></p> <p>(5) <math>(3x - 1)(x - 4)</math></p>	<p>(1) <math>(2x + 2y + 3)(x + y + 1)</math></p> <p>(2) <math>(2x + 2y + 3)(x + y - 1)</math></p> <p>(3) <math>(2x + 2y + 1)(x + y - 5)</math></p> <p>(4) <math>(3x + 3y - 1)(x + y - 4)</math></p>
13	14
<p>(6) <math>(5x + 1)(x + 3)</math></p> <p>(7) <math>(5x + 3y)(x - y)</math></p> <p>(8) <math>(2x - 3y)(x + 2y)</math></p> <p>(9) <math>(4x + 3)(x - 4)</math></p> <p>(10) <math>(3x + 4y)(2x - 3y)</math></p>	<p>(5) <math>(2x + 2y + 1)(x + y + 1)</math></p> <p>(6) <math>(2x + 2y + 5a)(x + y + a)</math></p> <p>(7) <math>(3x - 3y + 5a)(x - y - a)</math></p> <p>(8) <math>(7x + 7y - a)(x + y + 2a)</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	



15	16
<p>(1) <math>(3x - 2)^2</math></p> <p>(2) <math>(x + 2y)^2</math></p> <p>(3) <math>(2x + 3y)^2</math></p> <p>(4) <math>(x^2 - 4xy + 4y^2 - 3a(x - 2y)^2)</math></p> <p>(5) <math>-(x - 2)^2</math></p>	<p>(1) <math>(x - a + b)^2</math></p> <p>(2) <math>(x + 2y + 2z)^2</math></p> <p>(3) <math>(a + 3b - 3c)^2</math></p> <p>(4) <math>(a - 4b - 4c)^2</math></p> <p>(5) <math>(a - 6b + 9c)^2</math></p>
15	16
<p>(6) <math>(x + y + 2)^2</math></p> <p>(7) <math>(x + y + 4)^2</math></p> <p>(8) <math>(x + y - 5)^2</math></p> <p>(9) <math>(x - y - 3a)^2</math></p> <p>(10) <math>(x + 2y - z)^2</math></p>	<p>(6) <math>(4x + 9y)^2</math></p> <p>(7) <math>(x + y)^2</math></p> <p>(8) <math>(3x + 2y)^2</math></p> <p>(9) <math>(x - y)^2</math></p> <p>(10) <math>(7a + 4b)^2</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

J 17–20

17	18
<p>(1) <math>(3a + 2b)(3a - 2b)</math></p> <p>(2) <math>(x^2 - 4)</math> <math>5(x + 2)(x - 2)</math></p> <p>(3) <math>a(2x + 3y)(2x - 3y)</math></p> <p>(4) <math>(x + y + 2)(x + y - 2)</math></p> <p>(5) <math>(x + y + z)(x + y - z)</math></p>	<p>(1) <math>(a + b + c - d)(a + b - c + d)</math></p> <p>(2) <math>(a - b + c - d)(a - b - c + d)</math></p> <p>(3) <math>(a - b + c + d)(a - b - c - d)</math></p> <p>(4) <math>(x - y + a - b)(x - y - a + b)</math></p> <p>(5) <math>(4x + 5)(2x - 1)</math></p>
17	18
<p>(6) <math>(x + a + b)(x - a - b)</math></p> <p>(7) <math>(3 + x - 2y)(3 - x + 2y)</math></p> <p>(8) <math>(x + a - b)(x - a + b)</math></p> <p>(9) <math>(3a - b)(a + b)</math></p> <p>(10) <math>(a + 2b + c)(a + c)</math></p>	<p>(6) <math>5(a + b)(a - b)</math></p> <p>(7) <math>-5(a - b)(a + b)</math></p> <p>(8) <math>-3(x + y)(x - y)</math></p> <p>(9) <math>5(a - 1)(a - 3)</math></p> <p>(10) <math>4(x - 1)(x - 4)</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

19	20
<p>(1) <math>(x^2 + 4)(x + 2)(x - 2)</math></p> <p>(2) <math>(4x^2 + y^2)(2x + y)(2x - y)</math></p> <p>(3) <math>(x^4 + y^4)(x^2 + y^2)(x + y)(x - y)</math></p> <p>(4) <math>(x^4 + 1)(x^2 + 1)(x + 1)(x - 1)</math></p>	<p>(1) <math>(x + 2)^2(x - 2)^2</math></p> <p>(2) <math>(x + 3)^2(x - 3)^2</math></p> <p>(3) <math>(x + y)^2(x - y)^2</math></p> <p>(4) <math>x(x + 2)^2(x - 2)^2</math></p>
19	20
<p>(5) <math>(x^2 - 9)(x + 3)(x - 3)(x^2 + 4)</math></p> <p>(6) <math>(x + 3)(x - 3)(x^2 + 1)</math></p> <p>(7) <math>(x + 2)(x - 2)(x^2 - 2)</math></p> <p>(8) <math>(2x^2 + 3)(x + 1)(x - 1)</math></p> <p>(9) <math>a(x + 2)(x - 2)(x^2 + 1)</math></p>	<p>(5) <math>(x^2 + a^2)(x + a)(x - a)</math></p> <p>(6) <math>x^3(x^2 + 4)(x + 2)(x - 2)</math></p> <p>(7) <math>(x + 2y)^2(x - 2y)^2</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

J 21–24

21	22
<p>(1) <math>(2a - b)(x - y)</math></p> <p>(2) <math>3(a + 1)(x + 2)</math></p> <p>(3) <math>2(x + 3)(2a + 3)</math></p> <p>(4) <math>3(x - 3)(a + 1)</math></p> <p>(5) <math>3(a - 2)(x - 2)</math></p>	<p>(1) <math>(a - b)(3x + 2)(3x - 2)</math></p> <p>(2) <math>(a - b)(x + 4)(x - 4)</math></p> <p>(3) <math>(x - 2y)(2x + 3y)(2x - 3y)</math></p> <p>(4) <math>(x - 2y)(a + 2b)(a - 2b)</math></p> <p>(5) <math>(x - 3y)(a + 3)(a - 3)</math></p>
21	22
<p>(6) <math>(x - 2)(2 - a)</math>     <math>[ = -(x - 2)(a - 2) ]</math></p> <p>(7) <math>(a - b)(x + 2)</math></p> <p>(8) <math>(2a - b)(5x - 2y)</math></p> <p>(9) <math>a(2x - y)(3b + 5a)</math>     <math>[ = a(2x - y)(5a + 3b) ]</math></p> <p>(10) <math>2(x - 2y)(b + 2a)</math>     <math>[ = 2(x - 2y)(2a + b) ]</math></p>	<p>(6) <math>(a - b)(x - 2)(x + 1)</math></p> <p>(7) <math>(a - b)(x - 3)^2</math></p> <p>(8) <math>(a - b)(2x + 3)(x - 2)</math></p> <p>(9) <math>(a - b)(x - 5)(x + 2)</math></p> <p>(10) <math>(a - b)(2x + 1)(x + 3)</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

23	24
<p>(1) <math>(a - b)(3 + a - b)</math></p> <p>(2) <math>(a - b)(3 - a + b)</math></p> <p>(3) <math>(x - y)(x - y + 3)</math></p>	<p>(1) <math>(x - y)(-x + 3y)</math>  <math>[= -(x - y)(x - 3y)]</math></p> <p>(2) <math>x(x - 2y)(3x - 2y)</math></p> <p>(3) <math>2x(x - 3y)(x + 3y)</math></p> <p>(4) <math>2y(x - 2y)(2x - 3y)</math></p>
23	24
<p>(4) <math>2(x - 2y)(x - 2y + 2a)</math></p> <p>(5) <math>a(x - 2y)(x - 2y + 2a)</math></p> <p>(6) <math>x(2a - 3b)(2a - 3b - 2x)</math></p> <p>(7) <math>(x - y)(2 - x + y)</math></p> <p>(8) <math>a(x - 3y)(2a + x - 3y)</math></p>	<p>(5) <math>3a(a - 3b)(a - b)</math></p> <p>(6) <math>2(a - b)^2(x - 2y)</math></p> <p>(7) <math>3xy(x - y)(x - 2xy + 2y^2)</math></p> <p>(8) <math>5x(2y - 3x)(y - 3x)^2</math>  <math>[= -5x(3x - 2y)(3x - y)^2]</math></p>
<div data-bbox="356 1406 801 1481">NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

J 25–28

25	26
<p>(1) <math>(x - y)^2(1 - 3x + 3y)</math></p> <p>(2) <math>(a - b)^2(x - 2ay + 2by)</math></p> <p>(3) <math>2(x - 3)^2(2a^2 + x - 3)</math></p> <p>(4) <math>xy(x - 3y)^2(x - xy + 3y^2)</math></p>	<p>(1) <math>(x - y)(a + b)</math></p> <p>(2) <math>(x + y)(a - b)</math></p> <p>(3) <math>(x - y)(a - b)</math></p> <p>(4) <math>(b + c)(a + d)</math></p> <p>(5) <math>(b - c)(a + d)</math></p>
25	26
<p>(5) <math>3x(3y - x)(2y - x)(y - x)</math> [ <math>= -3x(x - 3y)(x - 2y)(x - y)</math> ]</p> <p>(6) <math>5xy(x - y)^2(x - 2y)(x + y)</math></p> <p>(7) <math>2x(2y - x)(3y - x)(y + x)</math> [ <math>= 2x(x - 2y)(x - 3y)(x + y)</math> ]</p>	<p>(6) <math>(b - c)(a - d)</math></p> <p>(7) <math>(b + d)(a + c)</math></p> <p>(8) <math>(a - d)(b - c)</math></p> <p>(9) <math>(a + d)(b - c)</math></p> <p>(10) <math>(a - d)(b + c)</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

27	28
<p>(1) <math>(a+x)(a+y)</math></p> <p>(2) <math>(a+x)(a-y)</math></p> <p>(3) <math>(a-x)(a+y)</math></p> <p>(4) <math>(x+y)(2a+3b)</math></p> <p>(5) <math>(x-y)(2a+3b)</math></p>	<p>(1) <math>(x-4)(x^2+2)</math></p> <p>(2) <math>(x+4)(x^2-2)</math></p> <p>(3) <math>(x^2-2)(x-4)</math></p> <p>(4) <math>(x+1)(x^2+1)</math></p> <p>(5) <math>(x-1)(x^2+1)</math></p>
27	28
<p>(6) <math>(a+2x)(a+b)</math></p> <p>(7) <math>(a+2x)(a-b)</math></p> <p>(8) <math>(a+b)(2x-a)</math></p> <p>(9) <math>(x+y)(3a+2b)</math></p> <p>(10) <math>(2x+y)(a-3b)</math></p>	<p>(6) <math>(x-2)(x^2+1)</math></p> <p>(7) <math>(x+1)(x+2)(x-2)</math></p> <p>(8) <math>(x+1)(x-1)^2</math></p> <p>(9) <math>(y+1)(y-1)(x^2+1)</math></p> <p>(10) <math>(y+1)(y-1)(x+1)(x-1)</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

J 29–32

29	30
<p>(1) <math>(x - y)(x - 1)</math></p> <p>(2) <math>(x + y)^2(x - y)</math></p> <p>(3) <math>(y + 1)(x + 1)</math></p> <p>(4) <math>(y + z)(x^2 + y^2)</math></p> <p>(5) <math>(x - 1)^2(x + 1)</math></p>	<p>(1) <math>x(x - 2y)(3x - 2y)</math></p> <p>(2) <math>2y(x - 2y)(2x - 3y)</math></p> <p>(3) <math>2(a - 2b)(x - 2y)</math></p> <p>(4) <math>2x(3y - x)(2y - x)(y - x)</math> [ <math>= -2x(x - 3y)(x - 2y)(x - y)</math> ]</p> <p>(5) <math>5x(2y - 3x)(y - 3x)^2</math> [ <math>= -5x(3x - 2y)(3x - y)^2</math> ]</p>
29	30
<p>(6) <math>(a - b)(a - b - x)</math></p> <p>(7) <math>(b - c)(a - b + c)</math></p> <p>(8) <math>(a - b)(a + b - c)</math></p> <p>(9) <math>(b + c)(a - b - c)</math></p>	<p>(6) <math>2(y - z)(x + 2z)</math></p> <p>(7) <math>3(2 - 3x^2)(1 + 2y)</math></p> <p>(8) <math>a(y + 1)(y - 1)(x + 1)(x - 1)</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	



31	32
<p>(1) <math>(x + a)(x + 3b)</math></p> <p>(2) <math>(x - 2a)(x - b)</math></p> <p>(3) <math>(x + 2a)(x - b)</math></p> <p>(4) <math>(x - 2a)(x + b)</math></p> <p>(5) <math>(x - a)(x + 3b)</math></p>	<p>(1) <math>(x + 2y + 3)(x + y + 1)</math></p> <p>(2) <math>(x + 2y + 3)(x + y + 2)</math></p> <p>(3) <math>(x - y - 6)(x - y + 1)</math></p> <p>(4) <math>(x - 2y - 3)(x - y - 1)</math></p>
31	32
<p>(6) <math>(x - a)(x + a + 3)</math></p> <p>(7) <math>(x - a)(x + a + b)</math></p> <p>(8) <math>(x + a)(x - a - b)</math></p> <p>(9) <math>(x + a)(x - a + 2)</math></p>	<p>(5) <math>(x - y - 6)(x + 2y - 1)</math></p> <p>(6) <math>(x - 2y - 3)(x + y + 1)</math></p> <p>(7) <math>(x + 2y + 3)(x - y - 2)</math></p> <p>(8) <math>(x + 2y - 1)(x - 3y + 5)</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

J 33–36

33	34
<p>(1) <math>(x + 2y + 3)(x + y + 2)</math></p> <p>(2) <math>(x - 2y - 5)(x - y + 1)</math></p> <p>(3) <math>(x - 3y + 5)(x - 2y + 1)</math></p>	<p>(1) <math>(x + 2y + 3)(x + y + 2)</math></p> <p>(2) <math>(x - 2y - 3)(x - y - 2)</math></p> <p>(3) <math>(x + 2y - 3)(x + y - 2)</math></p>
33	34
<p>(4) <math>(x + 3y - 1)(x - 2y + 1)</math></p> <p>(5) <math>(x - 2y - 3)(x + y - 1)</math></p> <p>(6) <math>(x - 2y - 3)(x + y + 4)</math></p> <p>(7) <math>(x - 3y + 2)(x + y - 3)</math></p> <p>(8) <math>(x - 3y + 1)(x + 2y - 1)</math></p>	<p>(4) <math>(x - 3y + 5)(x - 2y + 1)</math></p> <p>(5) <math>(x + 2y + 3)(x + y + 1)</math></p> <p>(6) <math>(x - 2y + 1)(x + y - 5)</math></p> <p>(7) <math>(x + 2y + 3)(x - y - 4)</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

35	36
<p>(1) <math>(x + 2y + 1)(x + y + 3)</math></p> <p>(2) <math>(x - 3y - 2)(x - 2y - 1)</math></p> <p>(3) <math>(x + 3y + 5)(x - y - 1)</math></p> <p>(4) <math>(x - 2y + 1)(x + y + 5)</math></p>	<p>(1) <math>(x + y + 1)^2</math></p> <p>(2) <math>(x - y - 1)^2</math></p> <p>(3) <math>(x - 2y - 2)^2</math></p> <p>(4) <math>(x - 3y - 3)^2</math></p>
35	36
<p>(5) <math>(x + 3y + 2)(x + y - 2)</math></p> <p>(6) <math>(x - 3y + 2)(x + 2y - 3)</math></p> <p>(7) <math>(x - 3y - 2)(x - y + 6)</math></p> <p>(8) <math>(x + 3y - 4)(x - 2y - 3)</math></p>	<p>(5) <math>(x + y + 3z)^2</math></p> <p>(6) <math>(x + 2y + z)^2</math></p> <p>(7) <math>(x + 2y + 3z)^2</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

J 37–40

37	38
<p>(1) <math>(x + 2y + 2)(x + y - 5)</math></p> <p>(2) <math>(x + 2y - 4)(x + y + 1)</math></p> <p>(3) <math>(x - 2y + 4)(x + y - 5)</math></p>	<p>(1) <math>(2x - a)(x - 3b)</math></p> <p>(2) <math>(2x + a)(x - 3b)</math></p> <p>(3) <math>(2x + 3b)(x + a)</math></p>
37	38
<p>(4) <math>(x + 3y + 3)(x + 2y + 2)</math></p> <p>(5) <math>(x - y - 1)(x + 6y + 6)</math></p> <p>(6) <math>(x - 2y - 2)(x + 6y + 6)</math></p>	<p>(4) <math>(2x - 3b)(x + a)</math></p> <p>(5) <math>(2x + 3b)(x - a)</math></p> <p>(6) <math>(2x + a)(x + a + 4)</math></p> <p>(7) <math>(2x - a)(x + a + 3)</math></p> <p>(8) <math>(2x + a + 4b)(x + a)</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

39	40
<p>(1) <math>(2x + y + 1)(x + 3y + 4)</math></p> <p>(2) <math>(2x + y - 1)(x + 3y - 4)</math></p> <p>(3) <math>(2x + 2y + 1)(x + 3y + 5)</math></p>	<p>(1) <math>(x + y + 2z)(x + 2y + z)</math></p> <p>(2) <math>(x - y - 2z)(x + 2y + z)</math></p> <p>(3) <math>(x + y + 2z)(x - 2y - z)</math></p> <p>(4) <math>(a - 2b - 3c)(a + b + c)</math></p>
39	40
<p>(4) <math>(3x + y + 2)(x + 2y + 3)</math></p> <p>(5) <math>(2x - y - 1)(x - 3y - 4)</math></p> <p>(6) <math>(2x - 2y - 1)(x - 3y - 5)</math></p> <p>(7) <math>(3x - y - 2)(x - 2y - 3)</math></p>	<p>(5) <math>(a + 4b + 2c)(a - 2b + c)</math></p> <p>(6) <math>(x - 2y + 1)(x + 2y + 2)</math></p> <p>(7) <math>(2x + a + 2b)(x - 3a + b)</math></p> <p>(8) <math>(2x - y + 1)(x - 3y + 4)</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

J 41–44

41	42
<p>(1) <math>(2x + y + 4)(x - 2y - 3)</math></p> <p>(2) <math>(2x - y - 4)(x + 2y + 3)</math></p> <p>(3) <math>(2x + y + 3)(x + 3y + 5)</math></p>	<p>(1) <math>(2a + b + c)(a + b + 2c)</math></p> <p>(2) <math>(2a - 2b - c)(a - b - 2c)</math></p>
41	42
<p>(4) <math>(2x - 3y + 1)(x - 2y + 3)</math></p> <p>(5) <math>(2x - y - 1)(x + y + 2)</math></p> <p>(6) <math>(2x - y + 1)(x + y + 1)</math></p> <p>(7) <math>(2x + y + 4)(x - 3y - 2)</math></p>	<p>(3) <math>(2a - b + 2c)(a + 3b - 2c)</math></p> <p>(4) <math>(3a + 2b - c)(a - 3b + 2c)</math></p> <p>(5) <math>(2a - b - c)(a + 2b + 3c)</math></p> <p>(6) <math>(2a + b - c)(a - 2b + c)</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

43	44
<p>1.</p> <p>(1) <math>(2b + a + 3c)(b + 3a + 2c)</math></p> <p>(2) <math>(2b - 2a + c)(b - a + 2c)</math></p>	<p>1.</p> <p>(1) <math>(a + 3b + c)(a + 3b - c)</math></p> <p>(2) <math>(2a - 3b + 2c)(2a - 3b - 2c)</math></p> <p>(3) <math>(2a + b - 3c)(2a - b + 3c)</math></p>
43	44
<p>2.</p> <p>(1) <math>(3a - 2b + 5)(a - b - 2)</math></p> <p>(2) <math>(2b - 3a - 5)(b - a + 2)</math></p>	<p>2.</p> <p>(1) <math>(a + b + x + 3)(a + b - x - 3)</math></p> <p>(2) <math>(b + a + x + 3)(b + a - x - 3)</math></p> <p>(3) <math>(a + b + x + 3)(a + b - x - 3)</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

J 45–48

45	46
<p>(1) <math>(x + y - 2)(x - y - 2)</math></p> <p>(2) <math>(x + y + 3)(x - y - 3)</math></p> <p>(3) <math>(2x - 3y + 3)(2x - 3y - 3)</math></p>	<p>(1) <math>(a + d + b - c)(a + d - b + c)</math></p> <p>(2) <math>(b - c + a - d)(b - c - a + d)</math></p> <p>(3) <math>(a - b)(a - b - 2c)</math></p>
45	46
<p>(4) <math>(x + y - 2a)(x - y + 2a)</math></p> <p>(5) <math>(x - 3a + y)(x - 3a - y)</math></p> <p>(6) <math>(3x + 2y - z)(3x - 2y + z)</math></p> <p>(7) <math>(x + 2y + z + 2)(x + 2y - z - 2)</math></p>	<p>(4) <math>(a - b)(x + 3)(x - 1)</math></p> <p>(5) <math>(2x - y - 1)(x + y + 2)</math></p> <p>(6) <math>(2x - y + 1)(x + y + 1)</math></p> <p>(7) <math>(x - 2y + 1)(x + 2y - 2)</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	



47	48
<p>(1) <math>(x+2y)(x^2-2xy+4y^2)</math></p> <p>(2) <math>(2x-3y)(4x^2+6xy+9y^2)</math></p> <p>(3) <math>(a-2b)(a^2+2ab+4b^2)</math></p> <p>(4) <math>(2a+3b)(4a^2-6ab+9b^2)</math></p> <p>(5) <math>(a-4)(a^2+4a+16)</math></p>	<p>(1) <math>(5a+2b)(25a^2-10ab+4b^2)</math></p> <p>(2) <math>(3x-4)(9x^2+12x+16)</math></p> <p>(3) <math>(x^3+8)</math>  <math>2(x+2)(x^2-2x+4)</math></p> <p>(4) <math>a(4a-3)(16a^2+12a+9)</math></p> <p>(5) <math>x(4-x)(16+4x+x^2)</math></p>
47	48
<p>(6) <math>(3a-2b)(9a^2+6ab+4b^2)</math></p> <p>(7) <math>(1-x)(1+x+x^2)</math></p> <p>(8) <math>(a+2b^2)(a^2-2ab^2+4b^4)</math></p> <p>(9) <math>(a-3b^3)(a^2+3ab^3+9b^6)</math></p> <p>(10) <math>(x^2+y^3)(x^4-x^2y^3+y^6)</math></p>	<p>(6) <math>(a-b)(a^2+4ab+7b^2)</math></p> <p>(7) <math>(2x+y)(4x^2+10xy+7y^2)</math></p> <p>(8) <math>a(a^2+3ab+3b^2)</math></p> <p>(9) <math>(a+c)(a^2+3b^2+c^2+3ab-3bc-ac)</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

J 49–52

49	50
<div>(1) <div>b</div> <div>b</div> ( a + b ) ( a - b ) ( a<sup>2</sup> + b<sup>2</sup> ) (a+b)(a-b)(a<sup>2</sup>+b<sup>2</sup>+ab)(a<sup>2</sup>+b<sup>2</sup>-ab)</div> <div>(2) <div>b<sup>3</sup></div> <div>b<sup>3</sup></div> (a+b)(a<sup>2</sup>-ab+b<sup>2</sup>)(a-b)(a<sup>2</sup>+ab+b<sup>2</sup>)</div> <div>(3) (a<sup>2</sup> + b<sup>2</sup>)(a<sup>4</sup> - a<sup>2</sup>b<sup>2</sup> + b<sup>4</sup>)</div> <div>(4) (a-b)(a<sup>2</sup>+ab+b<sup>2</sup>)(a<sup>6</sup>+a<sup>3</sup>b<sup>3</sup>+b<sup>6</sup>)</div> <div>(5) (a<sup>4</sup> + b<sup>4</sup>)(a<sup>8</sup> - a<sup>4</sup>b<sup>4</sup> + b<sup>8</sup>)</div>	<div>(1) (2x - y + 1)(x - 3y + 4)</div> <div>(2) (3x + y + 2)(x + 2y + 3)</div> <div>(3) (2x + y - z)(x - 2y + z)</div> <div>(4) (2a - 3b - 6c)(a - 2b + 3c)</div>
49	50
<div>(6) (x - y)(x + y)<sup>2</sup></div> <div>(7) (a + b)(a<sup>2</sup> - ab + b<sup>2</sup> + 1)</div> <div>(8) (a - b)<sup>3</sup></div> <div>(9) (2 + x)(2 - x)(1 + x)(1 - x + x<sup>2</sup>)</div>	<div>(5) (a + 6b + 6)(a - b - 1)</div> <div>(6) (a + 2b - 3c)(a - 2b + c)</div> <div>(7) (a + 2b - c)(a - 2b + c)</div> <div>(8) (1 + x)<sup>2</sup>(1 - x)(1 - x + x<sup>2</sup>)</div>
NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT	

51	52
<p>(1) <math>(x^2 + 3x - 1)(x^2 - 3x - 1)</math></p> <p>(2) <math>(x^2 + 5x - 1)(x^2 - 5x - 1)</math></p> <p>(3) <math>9x^2</math>  <math>(x^2 + 3x + 1)(x^2 - 3x + 1)</math></p>	<p>(1) <math>(x^2 + 4x - 1)(x^2 - 4x - 1)</math></p> <p>(2) <math>(x^2 + 4x + 1)(x^2 - 4x + 1)</math></p> <p>(3) <math>(x^2 + 6x - 1)(x^2 - 6x - 1)</math></p> <p>(4) <math>(x^2 + 6x + 1)(x^2 - 6x + 1)</math></p>
51	52
<p>(4) <math>(x^2 + 5x + 1)(x^2 - 5x + 1)</math></p> <p>(5) <math>(a^2 + 3a - 2)(a^2 - 3a - 2)</math></p> <p>(6) <math>(x^2 + 2x + 3)(x^2 - 2x + 3)</math></p> <p>(7) <math>(x^2 + x + 1)(x^2 - x + 1)</math></p> <p>(8) <math>(a^2 + ab + b^2)(a^2 - ab + b^2)</math></p>	<p>(5) <math>(2x^2 + x + 1)(2x^2 - x + 1)</math></p> <p>(6) <math>(2x^2 + x + 3)(2x^2 - x + 3)</math></p> <p>(7) <math>(x + 3)(x - 1)(x - 3)(x + 1)</math>  <math>[(x + 3)(x - 3)(x + 1)(x - 1)]</math></p> <p>(8) <math>(3x - 2)(x + 1)(3x + 2)(x - 1)</math></p>
<p>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</p>	

J 53–56

53	54
<p>(1) <math>(x^2 + x - 9)(x^2 + x - 3)</math></p> <p>(2) <math>(x^2 + 2x - 9)(x^2 + 2x + 2)</math></p> <p>(3) <math>(x^2 + 5x + 8)(x^2 + 5x - 2)</math></p>	<p>(1) <math>(x^2 - 5x + 9)(x^2 - 5x + 1)</math></p> <p>(2) <math>(x^2 + 5x + 7)(x^2 + 5x + 3)</math></p>
53	54
<p>(4) <math>(x^2 + 4x - 7)(x^2 - 3x - 7)</math></p> <p>(5) <math>(x^2 - 3x + 3)(x^2 + 2x + 3)</math></p> <p>(6) <math>(x - 6)(x + 3)(x^2 - x - 18)</math></p>	<p>(3) <math>(x^2 + x - 8)(x + 3)(x - 2)</math></p> <p>(4) <math>(x^2 + 3x + 5)(x^2 + 3x - 3)</math></p> <p>(5) <math>(x^2 + 5x + 5)^2</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

55	56
<p>(1) <math>(b - c)(a - b)(a - c)</math></p> <p>(2) <math>(y + z)(x + y)(x + z)</math></p>	<p>(1) <math>(b - c)(x + b)(x + c)</math></p> <p>(2) <math>-(b - c)(x - b)(x - c)</math> <math>[(c - b)(x - b)(x - c)]</math></p> <p>(3) <math>(y + z)(x - y)(x - z)</math></p>
55	56
<p>(3) <math>(b - c)(a + b)(a + c)</math></p> <p>(4) <math>(b + c)(a - b)(a - c)</math></p>	<p>(4) <math>(y + z)(x + y)(x + z)</math></p> <p>(5) <math>(b - c)(a - b)(a + c)</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

J 57–60

57	58
(1) $(x + y - 1)(x - y + 2z + 1)$	1. (1) $(b + 2c - 5)(a + 3b + 2)$  (2) $(a + 3b + 2)(b + 2c - 5)$
57	58
(2) $(x + y - 1)(x - y + 2z + 1)$	2. In powers of $a$ , $(a + 3b + 2)(a + b + 2c - 2)$  In powers of $b$ , $(a + 3b + 2)(a + b + 2c - 2)$  In powers of $c$ , $(a + 3b + 2)(a + b + 2c - 2)$
NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT	

59	60
<p>(1) <math>(x+1)(a+x+1)(a+x-1)</math> [<math>= (x+1)(x+a+1)(x+a-1)</math>]</p>	<p>(1) <math>(x-y-1)(x+y-z-3)</math> (2) <math>(a-b)(c+a+b)(c-a-b)</math> [<math>= -(a-b)(a+b+c)(a+b-c)</math>]</p>
59	60
<p>(2) <math>(a-b)(x+a+b)(x-a-b)</math></p>	<p>(3) <math>(b-c)(a+b)(a-c)</math> (4) <math>(xy+x-1)(xy-y-1)</math> [<math>(xy-y-1)(xy+x-1)</math>]</p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

J 61–64

61	62
<p>(1) <math>\frac{2b^3}{3a^2}</math></p> <p>(2) <math>\frac{4}{5x}</math></p> <p>(3) <math>\frac{1}{ax^2}</math></p> <p>(4) <math>\frac{3}{4a^2}</math></p> <p>(5) <math>-\frac{x^2}{3z^2}</math></p> <p>(6) <math>-\frac{2z}{y}</math></p>	<p>(1) <math>-\frac{x+7}{x+2}</math></p> <p>(2) <math>-\frac{a}{b}</math></p> <p>(3) <math>-\frac{x-5}{x+5}</math></p> <p>(4) <math>-\frac{m}{x-1}</math></p> <p>(5) <math>\frac{x-1}{x+1}</math></p>
61	62
<p>(7) <math>\frac{x-2}{x-4}</math></p> <p>(8) <math>\frac{x+4}{2x+3}</math></p> <p>(9) <math>\frac{1}{x-1}</math></p> <p>(10) <math>\frac{x-3}{x+2}</math></p> <p>(11) <math>\frac{3a}{a+2}</math></p>	<p>(6) <math>-\frac{x-y-z}{x+y-z}</math></p> <p>(7) <math>-\frac{a+b-c}{a+b+c}</math></p> <p>(8) <math>\frac{(x+y)^2}{x^2-xy+y^2}</math></p> <p>(9) <math>\frac{(x-y)^2}{x^2+xy+y^2}</math></p> <p>(10) <math>\frac{1}{a+b}</math></p>
<p>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</p>	



63	64
<p>(1) <math>\frac{a+b}{a-b}</math></p> <p>(2) <math>\frac{x+c}{x+b}</math></p> <p>(3) <math>\frac{1}{x-3}</math></p> <p>(4) <math>\frac{a}{a+b}</math></p>	<p>(1) <math>\frac{5b^2c^2}{12ax}</math></p> <p>(2) 1</p> <p>(3) <math>\frac{6ay}{bx}</math></p> <p>(4) <math>-\frac{x^2}{a^2y}</math></p> <p>(5) <math>-\frac{b^{11}}{8a^{11}}</math></p>
63	64
<p>(5) <math>\frac{a+c}{a^2+ab+b^2}</math></p> <p>(6) <math>\frac{x-y}{x^2-xy+y^2}</math></p> <p>(7) <math>\frac{x(x-y)}{x-2y}</math></p>	<p>(6) <math>\frac{z^3}{y^3}</math></p> <p>(7) <math>\frac{4a^2b}{7xy}</math></p> <p>(8) <math>-\frac{245a^7b^5}{9x^5y^3}</math></p> <p>(9) <math>-\frac{3b^2y}{2a^3x}</math></p>
<div data-bbox="356 1406 801 1481">NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

J 65–68

65	66
<p>(1) <math>2a(x+1)</math></p> <p>(2) <math>\frac{x^2}{(x+3)(x-2)}</math></p> <p>(3) <math>\frac{x-1}{y}</math></p> <p>(4) <math>(a+b)(a-b)</math></p>	<p>(1) <math>\frac{5}{12x}</math></p> <p>(2) <math>\frac{1}{6x}</math></p> <p>(3) <math>\frac{7}{12x}</math></p> <p>(4) <math>\frac{3b}{4a}</math></p> <p>(5) <math>\frac{23c}{12ab}</math></p>
65	66
<p>(5) <math>\frac{1}{ab}</math></p> <p>(6) <math>a</math></p> <p>(7) <math>\frac{1}{(x+1)(x+2)(x+3)}</math></p> <p>(8) <math>1</math></p>	<p>(6) <math>\frac{a+3b}{abx}</math></p> <p>(7) <math>\frac{8x+1}{x^2}</math></p> <p>(8) <math>\frac{1}{x^2}</math></p> <p>(9) <math>\frac{5x+6}{x(x+3)}</math></p> <p>(10) <math>\frac{4x}{(x-4)(x+4)}</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

67	68
$(1) -\frac{20x}{(x+5)(x-5)}$ $(2) \frac{3x+1}{x^2}$ $(3) -\frac{1}{(x+2)^2}$ $(4) \frac{2a+3c}{abc}$ $(5) \frac{5x+11}{(x+1)(x+2)(x+3)}$	$(1) \frac{4x^2+12}{(x-3)(x+3)(x-1)(x+1)}$ $\left[ = \frac{4(x^2+3)}{(x-3)(x+3)(x-1)(x+1)} \right]$
67	68
$(6) \frac{x^2}{x-1}$ $(7) \frac{x^2+4x+3}{(x-1)(x-3)}$ $\left[ = \frac{(x+3)(x+1)}{(x-1)(x-3)} \right]$ $(8) \frac{a^2+1}{(1+a)(1-a)}$ $\left[ = -\frac{a^2+1}{(a+1)(a-1)} \right]$	$(2) \frac{4a+10}{(a+1)(a+2)(a+3)(a+4)}$ $\left[ = \frac{2(2a+5)}{(a+1)(a+2)(a+3)(a+4)} \right]$ $(3) \frac{8}{(a^4-1)(a^4+1)}$ $\left[ = \frac{8}{a^8-1} \right]$
(9) 0	<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>

J 69–72

69	70
<p>(1) <math>\frac{1}{12}</math></p> <p>(2) <math>\frac{18}{5}</math></p> <p>(3) <math>\frac{x}{x+1}</math></p> <p>(4) <math>\frac{1}{x}</math></p>	<p>(1) <math>\frac{x+3}{x+2}</math></p> <p>(2) <math>\frac{a+b-c-d}{a-b+c-d}</math></p> <p>(3) <math>\frac{2a-1}{(a-1)(a+2)}</math></p>
69	70
<p>(5) <math>\frac{x}{x+1}</math></p> <p>(6) <math>\frac{x+2}{x+3}</math></p> <p>(7) <math>\frac{x+1}{x}</math></p> <p>(8) <math>\frac{x-7}{x-2}</math></p>	<p>(4) 0</p> <p>(5) <math>\frac{12}{x^2-4}</math></p> <p><math>\left[ = \frac{12}{(x+2)(x-2)} \right]</math></p> <p>(6) <math>\frac{6}{(x+1)(x+2)(x+3)}</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

71	72
<p>(1) <math>6\sqrt{2} - 6</math></p> <p>(2) <math>-\sqrt{6}</math></p> <p>(3) <math>9\sqrt{10}</math></p> <p>(4) <math>-6 + 6\sqrt{10}</math></p> <p>(5) <math>-5\sqrt{6}</math></p>	<p>(1) <math>-18</math></p> <p>(2) <math>2</math></p> <p>(3) <math>18</math></p> <p>(4) <math>-6</math></p> <p>(5) <math>-20</math></p>
71	72
<p>(6) <math>8 + 4\sqrt{3}</math></p> <p>(7) <math>9 - 6\sqrt{2}</math></p> <p>(8) <math>24 - 12\sqrt{3}</math></p> <p>(9) <math>18 + 12\sqrt{2}</math></p> <p>(10) <math>42 - 24\sqrt{3}</math></p>	<p>(6) <math>12 - 4\sqrt{5}</math></p> <p>(7) <math>-1 + \sqrt{2} + \sqrt{3}</math></p> <p>(8) <math>11 + 2\sqrt{6} + 4\sqrt{3} + 6\sqrt{2}</math></p> <p>(9) <math>11 - 2\sqrt{6} - 4\sqrt{3} + 6\sqrt{2}</math></p>

J 73–76

73	74
<p>(1) <math>-2\sqrt{6}</math></p> <p>(2) <math>-4+2\sqrt{10}</math></p> <p>(3) <math>-7+6\sqrt{2}</math></p>	<p>1.</p> <p>(1) <math>\frac{\sqrt{35}}{7}</math></p> <p>(2) <math>\frac{2\sqrt{5}}{5}</math></p> <p>(3) <math>\frac{\sqrt{6}}{6}</math></p> <p>(4) <math>\frac{\sqrt{3}}{3}</math></p> <p>(5) <math>\frac{\sqrt{5}}{10}</math></p> <p>(6) <math>\frac{5\sqrt{3}}{6}</math></p>
73	74
<p>(4) <math>-2\sqrt{6}</math></p> <p>(5) <math>-6+2\sqrt{15}</math></p> <p>(6) <math>2\sqrt{15}-2\sqrt{7}</math></p>	<p>2.</p> <p>(1) <math>-\frac{\sqrt{3}}{6}</math></p> <p>(2) <math>3\sqrt{6}</math></p> <p>(3) <math>\frac{4\sqrt{6}}{3}</math></p> <p>(4) <math>-\frac{\sqrt{2}}{2}</math></p> <p>(5) <math>2\sqrt{3}-2</math></p> <p>(6) <math>\frac{5\sqrt{6}}{12}</math></p>

75	76
<p>(1) <math>\frac{\sqrt{5}-1}{4}</math></p> <p>(2) <math>2\sqrt{3}-2</math></p> <p>(3) <math>\sqrt{5}-1</math></p> <p>(4) <math>\sqrt{3}-\sqrt{2}</math></p> <p>(5) <math>\sqrt{3}+\sqrt{2}</math></p>	<p>1.</p> <p>(1) <math>\sqrt{6}+2</math></p> <p>(2) <math>-3+2\sqrt{2}</math></p> <p>(3) <math>-2-2\sqrt{6}</math></p> <p>(4) <math>\frac{-\sqrt{3}+9}{3}</math></p> <p>(5) <math>-2\sqrt{6}-5</math></p>
75	76
<p>(6) <math>3-2\sqrt{2}</math></p> <p>(7) <math>4+\sqrt{15}</math></p> <p>(8) <math>4-\sqrt{15}</math></p> <p>(9) <math>\frac{11+3\sqrt{15}}{2}</math></p> <p>(10) <math>-\frac{11-3\sqrt{15}}{7}</math></p>	<p>2.</p> <p>(1) <math>5-2\sqrt{6}</math></p> <p>(2) <math>\frac{3+\sqrt{5}}{16}</math></p> <p>(3) <math>2\sqrt{3}-3</math></p> <p>(4) 10</p>

J 77–80

77	78
1. (1) $5 + 2\sqrt{6}$  (2) $7 + 2\sqrt{10}$  (3) $9 + 2\sqrt{14}$  (4) $3 + 2\sqrt{2}$  (5) $8 - 2\sqrt{15}$  (6) $7 - 2\sqrt{10}$	(1) $\sqrt{5} - \sqrt{2}$  (2) $\sqrt{7} - \sqrt{3}$  (3) $\sqrt{10} - \sqrt{5}$  (4) $\sqrt{5} - 1$  (5) $\sqrt{3} - \sqrt{2}$
77	78
2. (1) $\sqrt{7} + \sqrt{2}$  (2) $\sqrt{2} + 1$  (3) $\sqrt{5} + 1$  (4) $2 + \sqrt{2}$  (5) $\sqrt{7} + \sqrt{3}$	(6) $\sqrt{5} - 1$  (7) $2 - \sqrt{3}$  (8) $\sqrt{6} + \sqrt{2}$  (9) $3 - \sqrt{3}$  (10) $4 + \sqrt{3}$



79	80
<p>(1) <math>\frac{\sqrt{10} + \sqrt{2}}{2}</math></p> <p>(2) <math>\frac{\sqrt{14} - \sqrt{2}}{2}</math></p> <p>(3) <math>\frac{\sqrt{14} - \sqrt{6}}{2}</math></p> <p>(4) <math>\frac{3\sqrt{2} + \sqrt{10}}{2}</math></p>	<p>1.</p> <p>(1) <math>\sqrt{3} + 1</math></p> <p>(2) <math>\sqrt{5} - \sqrt{3}</math></p> <p>(3) <math>\frac{\sqrt{6} + \sqrt{2}}{2}</math></p> <p>2.</p> <p>(1) 14</p> <p>(2) <math>\sqrt{7}</math></p>
79	80
<p>(5) <math>5 - \sqrt{3}</math></p> <p>(6) <math>3 + \sqrt{5}</math></p> <p>(7) <math>2\sqrt{5}</math></p> <p>(8) <math>\frac{\sqrt{7} + 2}{3}</math></p>	<p>3.</p> <p>(1) <math>12 + 12\sqrt{3}</math></p> <p>(2) <math>\frac{9\sqrt{5}}{10}</math></p> <p>(3) <math>2\sqrt{3} - 3</math></p> <p>(4) 0</p>

J 81–84

81	82
<div>1.</div> <div>(1) 10</div> <div>(2) 22</div> <div>2.</div> <div>(1) <math>2\sqrt{5}</math></div> <div>(2) 3</div> <div>3.</div> <div>(1) <math>\frac{3}{2}</math></div> <div>(2) <math>\frac{1}{4}</math></div>	<div>1.</div> <div>(1) <math>a^2 - 2b</math></div> <div>(2) <math>a^4 - 4a^2b + 2b^2</math></div> <div>(3) <math>a^2 - 4b</math></div> <div>(4) <div>3</div><math>a^3 - 3ab</math></div>
81	82
<div>4.</div> <div>(1) <math>\sqrt{3}</math></div> <div>(2) <math>\frac{1}{2}</math></div> <div>5.</div> <div>(1) 12</div> <div>(2) 1</div>	<div>2.</div> <div>(1) 4</div> <div>(2) 1</div> <div>(3) 14</div> <div>(4) 194</div> <div>(5) 12</div> <div>(6) 52</div>

83	84
<p>1.</p> <p>(1) <math>2\sqrt{5}</math></p> <p>(2) 2</p> <p>(3) 16</p> <p>(4) 248</p> <p>(5) 12</p> <p>(6) <math>28\sqrt{5}</math></p>	<p>1.</p> <p>(1) 4</p> <p>(2) 1</p> <p>(3) 14</p> <p>(4) 52</p>
83	84
<p>2.</p> <p>(1) 8</p> <p>(2) 4</p> <p>(3) 56</p> <p>(4) 416</p> <p>(5) 14</p> <p>(6) <math>\frac{7}{2}</math></p>	<p>2.</p> <p>(1) 8</p> <p>(2) 1</p> <p>(3) 60</p> <p>(4) 488</p>

J 85–88

85	86
<p>1.</p> <p>(1) 1</p> <p>(2) -1</p> <p>(3) 1</p> <p>(4) 2</p>	<p>1.</p> <p>(1) <input type="text" value="4"/> and <input type="text" value="-4"/></p> <p>(2) <input type="text" value="5"/> and <input type="text" value="-5"/></p> <p>(3) <input type="text" value="5"/> and <input type="text" value="-5"/></p> <p>(4) <input type="text" value="5"/> and <input type="text" value="-5"/></p> <p>(5) <input type="text" value="6"/></p> <p>(6) <input type="text" value="-6"/></p> <p>(7) <input type="text" value="5"/></p> <p>(8) <input type="text" value="-5"/></p>
85	86
<p>2.</p> <p>(1) -1</p> <p>(2) <math>6\sqrt{3}</math></p> <p>(3) -5</p> <p>(4) <math>6\sqrt{3}</math></p>	<p>2.</p> <p>(1) <u>False/correct answer is 5</u></p> <p>(2) <u>False/correct answer is 5</u></p> <p>(3) <u>False/correct answer is 3</u></p> <p>(4) <u>True</u></p> <p>(5) <u>False/correct answer is 2</u></p> <p>(6) <u>True</u></p>

87	88
<p>1.</p> <p>(1) 2</p> <p>(2) 1</p> <p>(3) 0</p> <p>(4) 1</p> <p>(5) 2</p> <p>(6) 3</p> <p>(7) 3</p> <p>2.</p> <p>(1) <span style="border: 1px solid black; padding: 2px 10px;"><math>a</math></span></p> <p>(2) <span style="border: 1px solid black; padding: 2px 10px;"><math>-a</math></span></p>	<p>(1) <math>\begin{cases} x-2 &amp; (\text{when } x \geq 2) \\ -(x-2) &amp; (\text{when } x &lt; 2) \end{cases}</math></p> <p>(2) <math>\begin{cases} x-1 &amp; (\text{when } x \geq 1) \\ -(x-1) &amp; (\text{when } x &lt; 1) \end{cases}</math></p> <p>(3) <math>\begin{cases} x+1 &amp; (\text{when } x \geq -1) \\ -(x+1) &amp; (\text{when } x &lt; -1) \end{cases}</math></p> <p>(4) <math>\begin{cases} x &amp; (\text{when } x \geq 0) \\ -x &amp; (\text{when } x &lt; 0) \end{cases}</math></p>
87	88
<p>3.</p> <p>(1) 2</p> <p>(2) 1</p> <p>(3) 0</p> <p>(4) 1</p> <p>(5) 2</p> <p>(6) 3</p> <p>(7) 4</p>	<p>(5) <math>\begin{cases} x-4 &amp; (\text{when } x \geq 4) \\ -(x-4) &amp; (\text{when } x &lt; 4) \end{cases}</math></p> <p>(6) <math>\begin{cases} x-a &amp; (\text{when } x \geq a) \\ -(x-a) &amp; (\text{when } x &lt; a) \end{cases}</math></p> <p>(7) <math>\begin{cases} x+3 &amp; (\text{when } x \geq -3) \\ -(x+3) &amp; (\text{when } x &lt; -3) \end{cases}</math></p> <p>(8) <math>\begin{cases} x+a &amp; (\text{when } x \geq -a) \\ -(x+a) &amp; (\text{when } x &lt; -a) \end{cases}</math></p> <p>(9) <math>\begin{cases} x-2a &amp; (\text{when } x \geq 2a) \\ -(x-2a) &amp; (\text{when } x &lt; 2a) \end{cases}</math></p>
<div style="border: 1px solid black; border-radius: 10px; padding: 10px; text-align: center;"> <p>NOTE: ANY CONDITIONS FOR <math>x</math> (e.g. "when <math>x \geq 4</math>") MUST BE WRITTEN</p> </div>	

89	90
<p>1.</p> <p>(1) <math>x - 2</math></p> <p>(2) <math>x - 4</math></p> <p>2.</p> <p>(1) <math>\begin{cases} x - 3 &amp; (\text{when } x \geq 3) \\ -(x - 3) &amp; (\text{when } 2 &lt; x &lt; 3) \end{cases}</math></p> <p>(2) <math>\begin{cases} x - 4 &amp; (\text{when } x \geq 4) \\ -(x - 4) &amp; (\text{when } 2 &lt; x &lt; 4) \end{cases}</math></p>	<p>1.</p> <p>(1) 5</p> <p>(2) 1</p> <p>(3) 23</p> <p>(4) 110</p>
89	90
<p>3.</p> <p>(1) <math>x - 2</math></p> <p>(2) <math>x - 4</math></p> <p>(3) <math>\begin{cases} x - 6 &amp; (\text{when } x \geq 6) \\ -(x - 6) &amp; (\text{when } 4 &lt; x &lt; 6) \end{cases}</math></p> <p>4.</p> <p>(1) <math>-(x - 2)</math></p> <p>(2) <math>-(x + 2)</math></p> <p>(3) <math>\begin{cases} x + 5 &amp; (\text{when } -5 \leq x &lt; -3) \\ -(x + 5) &amp; (\text{when } x &lt; -5) \end{cases}</math></p>	<p>2.</p> <p>(1) <math>\begin{cases} x - 2 &amp; (\text{when } x \geq 2) \\ -(x - 2) &amp; (\text{when } x &lt; 2) \end{cases}</math></p> <p>(2) <math>\begin{cases} x + 5 &amp; (\text{when } x \geq -5) \\ -(x + 5) &amp; (\text{when } x &lt; -5) \end{cases}</math></p> <p>(3) <math>\begin{cases} x - a &amp; (\text{when } x \geq a) \\ -(x - a) &amp; (\text{when } x &lt; a) \end{cases}</math></p> <p>(4) <math>\begin{cases} x + 2a &amp; (\text{when } x \geq -2a) \\ -(x + 2a) &amp; (\text{when } x &lt; -2a) \end{cases}</math></p>
<div>NOTE: ANY CONDITIONS FOR <math>x</math> (e.g. "when <math>x \geq 4</math>") MUST BE WRITTEN</div>	

91	92
<p>(1) <math>x = -3, -2</math></p> <p>(3) <math>x = 8, -2</math></p> <p>(2) <math>x = 7, 5</math></p> <p>(4) <math>x = 2, -1</math></p>	<p>(1) <math>x = 5, 3</math></p> <p>(4) <math>x = 0, 3</math></p> <p>(2) <math>m = 5, -2</math></p> <p>(5) <math>x = 0, \frac{1}{3}</math></p> <p>(3) <math>a = \frac{1}{2}, 6</math></p> <p>(6) <math>x = 0, \frac{7}{2}</math></p>
91	92
<p>(5) <math>x = 2, 1</math></p> <p>(9) <math>x = 7, -5</math></p> <p>(6) <math>x = 3, -2</math></p> <p>(10) <math>x = -11, 9</math></p> <p>(7) <math>x = -\frac{2}{3}, \frac{6}{5}</math></p> <p>(11) <math>x = \frac{5}{2}, 6</math></p> <p>(8) <math>x = 10, -4</math></p> <p>(12) <math>x = \frac{2}{3}, -4</math></p>	<p>(7) <math>x = \frac{1}{2}, 2</math></p> <p>(10) <math>x = \frac{9}{4}, -2</math></p> <p>(8) <math>x = -4, 3</math></p> <p>(11) <math>x = 7a, -4a</math></p> <p>(9) <math>x = -21, 20</math></p> <p>(12) <math>x = -\frac{a}{2}, a</math></p>

J 93–96

93	94
<div><div>(1) <math>x = -\frac{3}{2}, \frac{3}{2}</math></div><div>(3) <math>x = \pm 4</math></div><div><math>\left[ x = \pm \frac{3}{2} \right]</math></div><div>(2) <math>x = \pm \frac{9}{2}</math></div><div>(4) <math>x = \pm 2\sqrt{3}</math></div></div>	<div><div>(1) <math>x = 1 \pm \sqrt{7}</math></div><div>(4) <math>x = -1, -5</math></div><div>(2) <math>x = -3 \pm \sqrt{5}</math></div><div>(5) <math>x = 10, 0</math></div><div>(3) <math>x = 3 \pm \sqrt{2}</math></div><div>(6) <math>x = 6, 0</math></div></div>
93	94
<div><div>(5) <math>x = \pm 5\sqrt{2}</math></div><div>(8) <math>x = \pm 3\sqrt{2}</math></div><div>(6) <math>x = \pm \frac{\sqrt{10}}{2}</math></div><div>(9) <math>x = \pm 1</math></div><div>(7) <math>x = \pm 17</math></div><div>(10) <math>x = \pm \sqrt{7}</math></div></div>	<div><div>(7) <math>x = 4, -1</math></div><div>(10) <math>x = \frac{3 \pm \sqrt{a}}{2}</math></div><div>(8) <math>x = -1, -\frac{7}{3}</math></div><div>(11) <math>x = \frac{-5 \pm \sqrt{b}}{a}</math></div><div>(9) <math>x = \frac{5}{2}, \frac{1}{2}</math></div><div>(12) <math>x = \frac{b \pm c}{a}</math></div></div>



95	96
<div> <div>(1)</div> <math display="block">x = 3 \pm 2\sqrt{3}</math> </div> <div> <div>(2)</div> <math display="block">x = 4 \pm \sqrt{11}</math> </div> <div> <div>(3)</div> <math display="block">x = -3 \pm \sqrt{3}</math> </div> <div> <div>(4)</div> <div> <math display="block">\boxed{a^2} \quad \boxed{a^2}</math> <math display="block">(x - a)</math> <math display="block">\boxed{a^2 - 1}</math> <math display="block">a \pm \sqrt{a^2 - 1}</math> </div> </div>	<div> <div>(2)</div> <math display="block">x = \frac{-3 \pm \sqrt{41}}{8}</math> </div> <div> <div>(1)</div> <math display="block">x = \frac{-5 \pm \sqrt{61}}{6}</math> </div> <div> <div>(3)</div> <math display="block">x = \frac{5 \pm \sqrt{73}}{8}</math> </div>
95	96
<div> <div>(5)</div> <math display="block">x = \frac{3 \pm \sqrt{37}}{2}</math> </div> <div> <div>(6)</div> <math display="block">x = \frac{-1 \pm \sqrt{5}}{2}</math> </div> <div> <div>(7)</div> <math display="block">x = \frac{-1 \pm \sqrt{33}}{8}</math> </div> <div> <div>(8)</div> <div> <math display="block">\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)</math> <math display="block">\left(x + \frac{a}{2}\right)</math> <math display="block">\boxed{a^2}</math> <math display="block">\frac{-a \pm \sqrt{16 + a^2}}{2}</math> </div> </div>	<div> <div>(5)</div> <math display="block">x = \frac{3}{2}, -\frac{1}{2}</math> </div> <div> <div>(4)</div> <math display="block">x = 1, -\frac{1}{3}</math> </div> <div> <div>(6)</div> <math display="block">x = a \pm \sqrt{b + a^2}</math> </div>

J 97–100

97	98
<div>1.</div> <div><div>(1) <math>x = 2, \frac{1}{3}</math></div><div>(3) <math>x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}</math></div><div>(2) <math>x = \frac{7 \pm \sqrt{37}}{6}</math></div></div>	<div><div>(1) <math>x = 2, \frac{1}{3}</math></div><div>(3) <math>x = \frac{8}{3}, -1</math></div><div>(2) <math>x = 1, -\frac{8}{3}</math></div><div>(4) <math>x = -\frac{1}{2}, -3</math></div></div>
97	98
<div>2.</div> <div><div>(2) <math>x = \frac{-7 \pm \sqrt{41}}{4}</math></div><div>(1) <div><div>-9</div><div><math>\frac{9 \pm \sqrt{57}}{6}</math></div></div></div><div>(3) <math>x = \frac{7 \pm \sqrt{57}}{4}</math></div></div>	<div><div>(5) <math>x = -1, -\frac{3}{2}</math></div><div>(8) <math>x = \frac{7 \pm \sqrt{41}}{4}</math></div><div>(6) <math>x = \frac{3}{2}, 1</math></div><div>(9) <math>x = \frac{9 \pm \sqrt{33}}{8}</math></div><div>(7) <math>x = \frac{-7 \pm \sqrt{57}}{4}</math></div><div>(10) <math>x = \frac{2}{3}, -3</math></div></div>

99	100
$\begin{array}{ll} (1) & (3) \\ x = -\frac{1}{2}, -3 & x = \frac{9 \pm \sqrt{113}}{8} \end{array}$ $\begin{array}{ll} (2) & (4) \\ x = 2, \frac{1}{2} & x = \frac{-7 \pm \sqrt{41}}{4} \end{array}$	$\begin{array}{ll} (1) & (4) \\ x = \frac{-9 \pm \sqrt{21}}{6} & x = \frac{9 \pm \sqrt{21}}{6} \end{array}$ $\begin{array}{ll} (2) & (5) \\ x = -\frac{1}{2}, -3 & x = \frac{5}{3}, 1 \end{array}$ $(3) \\ x = \frac{-7 \pm \sqrt{61}}{2}$
99	100
$\begin{array}{ll} (5) & (8) \\ x = \frac{-5 \pm \sqrt{13}}{6} & x = \frac{2}{3}, \frac{1}{4} \end{array}$ $\begin{array}{ll} (6) & (9) \\ x = -\frac{2}{3}, -1 & x = 2, \frac{1}{3} \end{array}$ $\begin{array}{ll} (7) & (10) \\ x = \frac{-5 \pm \sqrt{33}}{4} & x = \frac{-7 \pm \sqrt{109}}{6} \end{array}$	$\begin{array}{ll} (6) & (9) \\ x = 1, \frac{4}{9} & x = \frac{7 \pm 3\sqrt{5}}{2} \end{array}$ $\begin{array}{ll} (7) & (10) \\ x = \frac{-5 \pm \sqrt{37}}{2} & x = \frac{11 \pm \sqrt{37}}{6} \end{array}$ $(8) \\ x = -1, -\frac{5}{3}$

J 101–104

101	102
<div><div>(1) <math>x = \frac{-9 \pm \sqrt{57}}{6}</math></div><div>(2) <math>x = \frac{-7 \pm \sqrt{41}}{4}</math></div><div>(3) <math>x = \frac{-9 \pm \sqrt{33}}{8}</math></div><div>(4) <math>x = \frac{9 \pm \sqrt{57}}{6}</math></div><div>(5) <math>x = \frac{-7 \pm \sqrt{57}}{4}</math></div></div>	<div>1.<div><div>(1) <math>x = \frac{-7 \pm \sqrt{34}}{3}</math></div><div>(2) <math>x = -1, -\frac{5}{3}</math></div><div>(3) <math>x = 4 \pm \sqrt{11}</math></div><div>(4) <math>x = -3 \pm 2\sqrt{3}</math></div></div></div>
101	102
<div><div>(6) <math>x = -\frac{1}{3}, -2</math></div><div>(7) <math>x = 2, \frac{1}{3}</math></div><div>(8) <math>x = 1, -\frac{8}{3}</math></div><div>(9) <math>x = \frac{2}{3}, -3</math></div><div>(10) ( <math>2b'</math> )<div><div><math>b'^2 - ac</math></div><div><math>\frac{-b' \pm \sqrt{b'^2 - ac}}{a}</math></div></div></div></div>	<div>2.<div><div>(1) <math>x = \frac{-9 \pm \sqrt{65}}{8}</math></div><div>(2) <math>x = 3, \frac{1}{3}</math></div><div>(3) <math>x = -4 \pm \sqrt{19}</math></div><div>(4) <math>x = \frac{-7 \pm \sqrt{73}}{4}</math></div><div>(5) <math>x = \frac{-3 \pm \sqrt{23}}{2}</math></div><div>(6) <math>x = -\frac{1}{2}, -3</math></div></div></div>

103	104
$\begin{array}{ll} (1) & (3) \\ x = \frac{-7 \pm \sqrt{34}}{3} & x = -4 \pm \sqrt{19} \end{array}$ $\begin{array}{ll} (2) & (4) \\ x = \frac{-3 \pm \sqrt{3}}{2} & x = 2, -1 \end{array}$	$\begin{array}{ll} (1) & (4) \\ x = 6, 2 & x = 0, -2 \end{array}$ $\begin{array}{ll} (2) & (5) \\ x = 3 & x = -\frac{8}{3}, 1 \end{array}$ $\begin{array}{l} (3) \\ x = \frac{1}{3} \end{array}$
103	104
$\begin{array}{ll} (5) & (8) \\ x = \frac{2 \pm \sqrt{19}}{3} & x = 5 \pm 2\sqrt{6} \end{array}$ $\begin{array}{ll} (6) & (9) \\ x = \frac{4 \pm 2\sqrt{7}}{3} & x = -1, -\frac{5}{3} \end{array}$ $\begin{array}{ll} (7) & (10) \\ x = \frac{2}{3} & x = \frac{9 \pm \sqrt{33}}{6} \end{array}$	$\begin{array}{ll} (6) & (8) \\ x = \frac{-5 \pm \sqrt{73}}{6} & x = \frac{2 \pm \sqrt{7}}{3} \end{array}$ $\begin{array}{ll} (7) & (9) \\ x = \frac{1}{2}, 1 & x = -3, 2 \end{array}$

J 105–108

105	106
<div><div>(1) <math>x = \frac{7}{4}, 2</math></div><div>(4) <math>x = -\frac{9}{2}, \frac{1}{2}</math></div><div>(2) <math>x = -\frac{4}{5}, \frac{7}{3}</math></div><div>(5) <math>x = -8, 7</math></div><div>(3) <math>x = \frac{5}{3}, -7</math></div></div>	<div><div>(1) <math>x = -\frac{2}{3}, -\frac{3}{2}</math></div><div>(2) <math>x = \frac{1}{3}, -\frac{2}{5}</math></div></div>
105	106
<div><div>(6) <math>x = \frac{1 \pm \sqrt{7}}{3}</math></div><div>(8) <math>x = \frac{-2 \pm \sqrt{10}}{3}</math></div><div>(7) <math>x = 0, \frac{1}{3}</math></div><div>(9) <math>x = -\frac{1}{2}, -3</math></div></div>	<div><div>(3) <math>x = \frac{3}{2}, \frac{1}{2}</math></div><div>(4) <math>x = \frac{1}{2}, -\frac{5}{6}</math></div></div>

107	108
(1) $(4x+3)(3x+4)$	(1) $(x-3\sqrt{2})(x+2\sqrt{2})$  (2) $(x-2\sqrt{3})(x-1)$
107	108
(2) $(3x-4)(5x+6)$  (3) $(x-3)(3x+16)$	(3) $(x-a)(x+2b)$  (4) $(x-a-b)(x-a+b)$
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	

J 109–112

109	110
1. $b = 2, \ c = -1$ $x = -1 - \sqrt{2}$	1. (1) $x = \frac{3}{7}, -5$ (4) $x = -\frac{3}{2}, 4$  (2) $x = \frac{1 \pm \sqrt{13}}{2}$ (5) $x = 0, \frac{11}{2}$  (3) $x = -2 \pm \sqrt{3}$ (6) $x = 2 \pm \sqrt{7}$
109	110
2. $a = 1, \ b = -4$ $x = 2 + \sqrt{2}$	2. $a = -18$ $x = -3$  3. $a = -8$ $x = -3 - \sqrt{17}$



111	112
<p>1.</p> <p>(1) 4                      (6) 4</p> <p>(2) 25                    (7) 25</p> <p>(3) <math>\frac{1}{4}</math>                    (8) <math>\frac{1}{4}</math></p> <p>(4) 2                      (9) 2</p> <p>(5) 3                      (10) 3</p>	<p>(1) <math>-9</math></p> <p>(2) <math>-9</math></p> <p>(3) <math>-16</math></p> <p>(4) <math>-49</math></p> <p>(5) <math>-3</math></p> <p>(6) <math>-3</math></p> <p>(7) <math>-5</math></p> <p>(8) <math>-8</math></p>
111	112
<p>2.</p> <p>(1) <math>\sqrt{3}i</math></p> <p>(2) <math>\sqrt{6}i</math></p> <p>(3) <math>3i</math></p> <p>(4) <math>2\sqrt{3}i</math></p> <p>(5) <math>3\sqrt{2}i</math></p> <p>(6) <math>\frac{1}{5}i</math></p> <p>(7) <math>\frac{\sqrt{3}}{4}i</math></p>	<p>(9) <math>-12</math></p> <p>(10) 12</p> <p>(11) <math>-2\sqrt{3}</math></p> <p>(12) <math>4\sqrt{3}</math></p> <p>(13) <math>-12</math></p> <p>(14) <math>-6\sqrt{6}</math></p> <p>(15) <math>-6</math></p> <p>(16) 6</p>

J 113–116

113	114
<p>(1) <math>\frac{3}{4}i</math></p> <p>(2) <math>-\frac{3}{4}i</math></p> <p>(3) <math>\frac{3}{5}i</math></p> <p>(4) <math>-\frac{3}{5}i</math></p> <p>(5) <math>-\frac{3}{5}i</math></p> <p>(6) <math>\frac{3}{5}i</math></p>	<p>(1) <math>4+5\sqrt{2}i</math></p> <p>(2) <math>46-7\sqrt{2}i</math></p> <p>(3) 18</p> <p>(4) <math>1+4\sqrt{3}i</math></p> <p>(5) <math>-4+2\sqrt{5}i</math></p> <p>(6) <math>1+12\sqrt{2}i</math></p>
113	114
<p>(7) <math>8+i</math></p> <p>(8) <math>9+7i</math></p> <p>(9) <math>4+5\sqrt{2}i</math></p> <p>(10) <math>23-\sqrt{3}i</math></p> <p>(11) 13</p> <p>(12) <math>5+12i</math></p>	<p>(7) <math>\frac{5+i}{13}</math></p> <p>(8) <math>1+i</math></p> <p>(9) <math>i</math></p> <p>(10) <math>\frac{3-4i}{5}</math></p> <p>(11) <math>\frac{4}{5}</math></p> <p>(12) <math>-\frac{2}{5}</math></p>

115	116
<p>1.</p> <p>(1) <math>i</math></p> <p>(2) <math>-1</math></p> <p>(3) <math>-i</math></p> <p>(4) <math>1</math></p> <p>(5) <math>-i</math></p> <p>(6) <math>-1</math></p> <p>(7) <math>i</math></p> <p>(8) <math>1</math></p>	<p>(1) <math>x = \frac{-5 \pm \sqrt{7} i}{4}</math></p> <p>(2) <math>x = \frac{1 \pm 2i}{2}</math></p>
115	116
<p>2.</p> <p>(1) <math>x=4, y=7</math></p> <p>(2) <math>(x-y)(x+2y)</math> <math>x=2, y=1</math></p>	<p>(3) <math>x = \frac{4 \pm \sqrt{5} i}{3}</math></p> <p>(6) <math>x = \pm 2</math></p> <p>(4) <math>x = \frac{-4 \pm \sqrt{2} i}{2}</math></p> <p>(7) <math>x = \frac{5}{2}, 0</math></p> <p>(5) <math>x = \frac{3}{2}</math></p> <p>(8) <math>x = 2 \pm i</math></p>

J 117-120

117	118
<div><div>(1) <math>x = \frac{3 \pm \sqrt{11} \, i}{5}</math></div><div>(4) <math>x = \frac{3 \pm \sqrt{11} \, i}{2}</math></div></div> <div><div>(2) <math>x = -\frac{2}{3}</math></div><div>(5) <math>x = \frac{4 \pm \sqrt{5} \, i}{3}</math></div></div> <div><div>(3) <math>x = \frac{-5 \pm \sqrt{15} \, i}{4}</math></div></div>	<div><div>(1) <math>x = -6, -2</math></div><div>(3) <math>x = \frac{1 \pm \sqrt{7}}{3}</math></div></div> <div><div>(2) <math>x = -\frac{5}{2}, -\frac{3}{2}</math></div><div>(4) <math>x = \frac{3 \pm \sqrt{39} \, i}{8}</math></div></div>
117	118
<div><div>(6) <math>x = \frac{1}{2}, 4</math></div><div>(8) <math>x = \frac{\sqrt{3} \pm \sqrt{11}}{2}</math></div></div> <div><div>(7) <math>x = 0, \frac{11}{2}</math></div><div>(9) <math>x = \sqrt{5}, -2\sqrt{5}</math></div></div>	<div><div>(5) <math>x = \frac{10 \pm 2\sqrt{2} \, i}{3}</math></div><div>(8) <math>x = 3a, a</math></div></div> <div><div>(6) <math>x = \frac{5 \pm \sqrt{7} \, i}{4}</math></div><div>(9) <math>x = a + b, a - b</math> <math>[x = a \pm b]</math></div></div> <div><div>(7) <math>x = m \pm \sqrt{m^2 - n^2}</math></div></div>

119	120
<p>1.</p> <p>(1) <math>(x-1-2i)(x-1+2i)</math></p> <hr/> <p>(2) <math>(x-i)(x+i)</math></p>	<p>1.</p> <p>(1) <math>x = 6 \pm 2\sqrt{11}</math></p> <p>(4) <math>x = a \pm ai</math></p> <p>(2) <math>x = -\frac{7}{3}, -\frac{5}{3}</math></p> <p>(5) <math>x = -1, -2a+1</math></p> <p>(3) <math>x = \frac{-1 \pm 2i}{5}</math></p>
119	120
<p>2.</p> <p><math>a = 2, b = 2, x = 1 - i</math></p>	<p>2.</p> <p>(1) <math>-\sqrt{15}</math></p> <p>(2) <math>\sqrt{15}</math></p> <p>(3) <math>-\frac{\sqrt{15}}{5}i</math></p> <p>(4) <math>\frac{\sqrt{15}}{5}i</math></p> <p>(5) <math>27 - 8i</math></p> <p>(6) <math>7 - 6\sqrt{2}i</math></p> <p>(7) <math>1 + i</math></p>

J 121–124

121	122
<div>1.</div> <div><div>(1)</div><div><math>x = -3, -5</math></div></div> <div><div>(4)</div><div><math>x = 2</math></div></div> <div><div>(2)</div><div><math>x = -4</math></div></div> <div><div>(5)</div><div><math>x = 2 \pm i</math></div></div> <div><div>(3)</div><div><math>x = -4 \pm i</math></div></div> <div><div>(6)</div><div><math>x = 2 \pm \sqrt{2}</math></div></div> <div><div>( 6 )</div></div> <div><div>( 4 )</div></div> <div><div>( 5 )</div></div>	<div>(1) there are 2 different real solutions.</div> <div>(2) there are 2 different complex solutions.</div> <div>.....</div> <div>(3) there is a repeated real solution.</div> <div>(4) there are 2 different real solutions.</div>
121	122
<div>2.</div> <div><div>(1)</div><div><math>\frac{5 \pm \sqrt{5}}{2}</math></div><div>real</div></div> <div><div>(2)</div><div>5</div><div>repeated</div><div>0</div></div> <div><div>(3)</div><div><math>\frac{5 \pm \sqrt{3} i}{2}</math></div><div>complex</div><div>-3</div></div>	<div>(5) there are 2 different complex solutions.</div> <div>(6) there is a repeated real solution.</div> <div>(7) there are 2 different complex solutions.</div> <div>.....</div> <div>(8) there are 2 different complex solutions.</div> <div>(9) there is a repeated real solution.</div> <div>(10) there are 2 different complex solutions.</div>

123	124
<p>(1) there is a repeated real solution.</p> <p>(2) there are 2 different real solutions.</p> <hr/> <p>(3) there is a repeated real solution.</p> <p>(4) there are 2 different real solutions.</p>	<p>(1) <math>k = \frac{25}{12}</math>      (3) <math>k = 4</math></p> <p>(2) <math>k = \frac{16}{3}</math>      (4) <math>k = 3</math></p>
123	124
<p>(5) there are 2 different complex solutions.</p> <p>(6) there are 2 different real solutions.</p> <p>(7) there are 2 different complex solutions.</p> <hr/> <p>(8) there are 2 different complex solutions.</p> <p>(9) there is a repeated real solution.</p> <p>(10) there are 2 different complex solutions.</p>	<p>(5) <math>k = 1</math>      (7) <math>k = \frac{2 \pm \sqrt{3}}{2}</math></p> <p>(6) <math>k = 0, 3</math>      (8) <math>k = \frac{-5 \pm 2\sqrt{5}}{5}</math></p>

J 125–128

125	126
<div><div>(1) <math>a \leq \frac{4}{3}</math></div><div>(3) <math>a \leq 0</math></div></div> <div><div>(2) <math>a \leq \frac{61}{24}</math></div><div>(4) <math>a \geq \frac{1}{4}</math></div></div>	<div><div>(1) <math>k &gt; \frac{1}{3}</math></div><div>(3) <math>k &gt; 1</math></div></div> <div><div>(2) <math>k &gt; \frac{17}{8}</math></div><div>(4) <math>k &lt; -\frac{1}{12}</math></div></div>
125	126
<div><div>(5) <math>a \leq \frac{3}{2}</math></div><div>(7) <math>a \leq -\frac{1}{2}</math></div></div> <div><div>(6) <math>a \leq -\frac{3}{4}</math></div><div>(8) <math>a &gt; -1</math></div></div>	<div><div>(5) <math>k &gt; \frac{1}{2}</math></div><div>(7) <math>k &gt; -\frac{1}{4}</math></div></div> <div><div>(6) <math>k &lt; -\frac{5}{4}</math></div><div>(8) <math>k &lt; -\frac{1}{3}</math></div></div>



127	128
<p>(1)</p> $\begin{cases} \text{When } k < \frac{9}{4}, \text{ there are 2 different real solutions.} \\ \text{When } k = \frac{9}{4}, \text{ there is a repeated real solution.} \\ \text{When } k > \frac{9}{4}, \text{ there are 2 different complex solutions.} \end{cases}$	<p>(1) there are 2 different real solutions.</p> <p>(2) there are 2 different real solutions.</p> <p>(3) there are 2 different complex solutions.</p>
127	128
<p>(2)</p> $\begin{cases} \text{When } a < \frac{1}{3}, \text{ there are 2 different real solutions.} \\ \text{When } a = \frac{1}{3}, \text{ there is a repeated real solution.} \\ \text{When } a > \frac{1}{3}, \text{ there are 2 different complex solutions.} \end{cases}$ <p>(3)</p> $\begin{cases} \text{When } k > -\frac{9}{8}, \text{ there are 2 different real solutions.} \\ \text{When } k = -\frac{9}{8}, \text{ there is a repeated real solution.} \\ \text{When } k < -\frac{9}{8}, \text{ there are 2 different complex solutions.} \end{cases}$ <p>(4)</p> $\begin{cases} \text{When } m > -\frac{2}{3}, \text{ there are 2 different real solutions.} \\ \text{When } m = -\frac{2}{3}, \text{ there is a repeated real solution.} \\ \text{When } m < -\frac{2}{3}, \text{ there are 2 different complex solutions.} \end{cases}$	<p>(4) <math>\begin{cases} \text{When } a = 0, \\ \text{there is a repeated real solution.} \\ \text{When } a \neq 0, \\ \text{there are 2 different real solutions.} \end{cases}</math></p> <p>(5) <math>\begin{cases} \text{When } a = 0, \\ \text{there is a repeated real solution.} \\ \text{When } a \neq 0, \\ \text{there are 2 different complex solutions.} \end{cases}</math></p> <p>(6) there is a repeated real solution for all real values of <math>a</math>.</p>

129	130
<p>1.</p> <p>When <math>m = 0</math>, <math>x = 0</math></p> <p>When <math>m = \frac{3}{4}</math>, <math>x = -\frac{1}{2}</math></p>	<p>1.</p> <p>(1)</p> <p><math>\left\{ \begin{array}{l} \text{When } a &lt; \frac{13}{4}, \text{ there are 2 different real solutions.} \\ \text{When } a = \frac{13}{4}, \text{ there is a repeated real solution.} \\ \text{When } a &gt; \frac{13}{4}, \text{ there are 2 different complex solutions.} \end{array} \right.</math></p> <p>(2)</p> <p><math>\left\{ \begin{array}{l} \text{When } a = b \text{ there is a repeated real solution.} \\ \text{When } a \neq b \text{ there are 2 different real solutions.} \end{array} \right.</math></p>
129	130
<p>2.</p> <p><math>x = -3</math></p>	<p>2.</p> <p><math>D = (a + c - 2b)^2 = 0</math></p> <p><math>a + c - 2b = 0</math></p> <p>Thus,</p> <p><math>b = \frac{a + c}{2}</math></p>

131	132
<p>1.</p> <p>(1) <math>\alpha + \beta = 2</math> <math>\alpha\beta = -8</math></p> <p>(2) <math>\alpha + \beta = 7</math> <math>\alpha\beta = 5</math></p> <p>(3) <math>\alpha + \beta = -\frac{2}{3}</math> <math>\alpha\beta = \frac{5}{3}</math></p> <p>(4) <math>\alpha + \beta = -\frac{b}{a}</math> <math>\alpha\beta = \frac{c}{a}</math></p>	<p>1.</p> <p>(1) <math>\alpha + \beta = -\frac{7}{3}</math> <math>\alpha\beta = -2</math></p> <p>(2) <math>\alpha + \beta = 0</math> <math>\alpha\beta = 3</math></p> <p>(3) <math>\alpha + \beta = \frac{3}{5}</math> <math>\alpha\beta = 0</math></p> <p>(4) <math>\alpha + \beta = -a</math> <math>\alpha\beta = b</math></p> <p>(5) <math>\alpha + \beta = -\frac{b}{a}</math> <math>\alpha\beta = \frac{c}{a}</math></p> <p>(6) <math>\alpha + \beta = \frac{5}{6}</math> <math>\alpha\beta = \frac{1}{6}</math></p>
131	132
<p>2.</p> <p>(1) <math>\alpha + \beta = \frac{7}{3}</math> <math>\alpha\beta = 2</math></p> <p>(2) <math>\alpha + \beta = -2</math> <math>\alpha\beta = \frac{7}{3}</math></p> <p>(3) <math>\alpha + \beta = \frac{6}{7}</math> <math>\alpha\beta = \frac{1}{7}</math></p>	<p>2.</p> <p>(1) <math>12x^2 - 5x - 2 = 0</math></p> <p>(2) <math>3x^2 - 4x - 4 = 0</math></p> <hr/> <p>(3) <math>x^2 + 2x - 15 = 0</math></p> <p>(4) <math>x^2 + 11x + 24 = 0</math></p>

J 133–136

133	134
<div>(1) <math>x^2 - 6x + 4 = 0</math></div> <div>(2) <math>x^2 - 3 = 0</math></div> <div>.....</div> <div>(3) <math>x^2 - 4x + 7 = 0</math></div> <div>(4) <math>x^2 - 4x + 13 = 0</math></div> <div>(5) <math>x^2 - 2x + 2 = 0</math></div>	<div>1.</div> <div>(1) <math>-4</math></div> <div>(2) <math>20</math></div> <div>(3) <math>11</math></div>
133	134
<div>(6) <math>x^2 - x - 1 = 0</math></div> <div>(7) <math>x^2 - (a + b)x + ab = 0</math></div> <div>.....</div> <div>(8) <math>x^2 - 2ax + a^2 + b^2 = 0</math></div> <div>(9) <math>x^2 - 6ax + 9a^2 - 20b^2 = 0</math></div>	<div>2.</div> <div>(1) <math>3</math></div> <div>(2) <math>-5</math></div> <div>(3) <math>-3</math></div>

135	136
<p>1.</p> <p>(1) <math>\frac{5}{3}</math></p> <p>(2) <math>-2</math></p> <p>(3) <math>\frac{23}{9}</math></p>	<p>1.</p> $x^2 - \frac{4}{7}x + \frac{1}{7} = 0$
135	136
<p>2.</p> <p>(1) 27</p> <p>(2) 140</p> <p>(3) 727</p>	<p>2.</p> $x^2 - x + \frac{1}{3} = 0$

J 137–140

137	138
<p>1.</p> $x^2 - 5x + 5 = 0$	<p>1.</p> <p>When <math>\alpha = -4</math>, <math>k = -64</math></p> <p>When <math>\alpha = 3</math>, <math>k = 27</math></p>
137	138
<p>2.</p> $a = 2, \quad b = -5$	<p>2.</p> $\alpha = 4, \quad k = 32$ <p>3.</p> <p>When <math>\alpha = \sqrt{6}</math>, <math>k = -3\sqrt{6}</math></p> <p>When <math>\alpha = -\sqrt{6}</math>, <math>k = 3\sqrt{6}</math></p>

139	140
<p>1.</p> $0 < k < \frac{8}{3}$	<p>1.</p> <p>(1) <math>9x^2 - 12x - 1 = 0</math></p> <p>(2) <math>x^2 + x + 1 = 0</math></p> <p>2.</p> <p>(1) 28</p> <p>(2) 59</p>
139	140
<p>2.</p> $0 < m < 1$	<p>3.</p> <p>(1) <math>\alpha + \beta = -2m</math>  <math>\alpha\beta = m^2 - 2m + 3</math></p> <p>(2) <math>(\alpha - \beta)^2 = 8m - 12</math></p> <p>(3) <math>m = 2</math></p>

J 141–144

141	142
$(1) \begin{cases} x = -\frac{6}{5} \\ y = -\frac{17}{5} \end{cases} \quad \begin{cases} x = 2 \\ y = 3 \end{cases}$	$(1) \begin{cases} x = -\frac{10}{3} \\ y = -\frac{17}{3} \end{cases} \quad \begin{cases} x = 2 \\ y = 5 \end{cases}$ $(2) \begin{cases} x = \frac{1}{7} \\ y = -\frac{5}{7} \end{cases} \quad \begin{cases} x = 1 \\ y = 1 \end{cases}$
141	142
$(2) \begin{cases} x = -2 \\ y = 1 \end{cases} \quad \begin{cases} x = -1 \\ y = 2 \end{cases}$ $(3) \begin{cases} x = -1 \\ y = -2 \end{cases} \quad \begin{cases} x = 2 \\ y = 1 \end{cases}$	$(3) \begin{cases} x = 4 \\ y = 1 \end{cases}$ $(4) \begin{cases} x = 2 \\ y = 1 \end{cases}$



143

$$(1) \begin{cases} x = \pm \boxed{4} \\ y = \pm \boxed{6} \end{cases}$$

144

$$(1) \begin{cases} x = 0 \\ y = 1 \end{cases} \quad \begin{cases} x = 1 \\ y = 3 \end{cases}$$

143

$$(2) \begin{cases} x = \frac{-1 \pm \sqrt{5}}{2} \\ y = \frac{1 \pm \sqrt{5}}{2} \end{cases}$$

$$(3) \begin{cases} x = \frac{2 \pm \sqrt{10}}{3} \\ y = \frac{1 \pm 2\sqrt{10}}{3} \end{cases}$$

144

$$(2) \begin{cases} x = -3 \\ y = -4 \end{cases} \quad \begin{cases} x = 2 \\ y = 1 \end{cases}$$

$$(3) \begin{cases} x = \frac{-2 \pm \sqrt{7}}{3} \\ y = 2 \mp \sqrt{7} \end{cases}$$

J 145–148

145	146
$(1) \begin{cases} x = \pm \sqrt{5} \\ y = \mp \sqrt{5} \end{cases}$	$(1) \begin{cases} x = \pm \sqrt{6} \\ y = \pm \frac{\sqrt{6}}{3} \end{cases} \quad \begin{cases} x = \pm 2 \\ y = \pm 1 \end{cases}$
145	146
$(2) \begin{cases} x = \frac{9}{5} \\ y = -\frac{3}{5} \end{cases} \quad \begin{cases} x = -3 \\ y = 1 \end{cases}$ $(3) \begin{cases} x = 2 \\ y = -1 \end{cases} \quad \begin{cases} x = 1 \\ y = 1 \end{cases}$	$(2) \begin{cases} x = \pm 2 \\ y = \mp 1 \end{cases}$

147	148
$(1) \begin{cases} x = 2 + \sqrt{2} \\ y = 2 - \sqrt{2} \end{cases} \quad \begin{cases} x = 2 - \sqrt{2} \\ y = 2 + \sqrt{2} \end{cases}$	$(1) \begin{cases} x = 2 + \sqrt{6} \\ y = 2 - \sqrt{6} \end{cases} \quad \begin{cases} x = 2 - \sqrt{6} \\ y = 2 + \sqrt{6} \end{cases}$
147	148
$(2) \begin{cases} x = 6 \\ y = -1 \end{cases} \quad \begin{cases} x = -1 \\ y = 6 \end{cases}$  $(3) \begin{cases} x = 1 \\ y = \sqrt{3} \end{cases} \quad \begin{cases} x = \sqrt{3} \\ y = 1 \end{cases}$	$(2) \begin{cases} x = 3 \\ y = 1 \end{cases} \begin{cases} x = 1 \\ y = 3 \end{cases} \begin{cases} x = -3 \\ y = -1 \end{cases} \begin{cases} x = -1 \\ y = -3 \end{cases}$  $(3) \begin{cases} x = \sqrt{2} \\ y = \sqrt{2} \end{cases} \quad \begin{cases} x = -\sqrt{2} \\ y = -\sqrt{2} \end{cases}$

# J 149–152

149	150
<p>(1) ( 5 - A )</p> <p><input type="text" value="7"/> <input type="text" value="12"/></p> <p>( A - 4 )( A - 3 )</p> <p>A = <input type="text" value="4"/> B = <input type="text" value="1"/></p> <p>A = <input type="text" value="3"/> B = <input type="text" value="2"/></p> <p>(i) <input type="text" value="4"/> <input type="text" value="1"/> (ii) <input type="text" value="3"/> <input type="text" value="2"/></p> <p><math>\begin{cases} x = 2 + \sqrt{3} \\ y = 2 - \sqrt{3} \end{cases}</math> <math>\begin{cases} x = 2 - \sqrt{3} \\ y = 2 + \sqrt{3} \end{cases}</math></p> <p><math>\begin{cases} x = 2 \\ y = 1 \end{cases}</math> <math>\begin{cases} x = 1 \\ y = 2 \end{cases}</math></p>	<p>(1) <math>\begin{cases} x = 0 \\ y = -3 \end{cases}</math> <math>\begin{cases} x = 3 \\ y = 0 \end{cases}</math></p> <p>(2) <math>\begin{cases} x = -1 \\ y = -2 \end{cases}</math> <math>\begin{cases} x = 2 \\ y = 1 \end{cases}</math></p>
149	150
<p>(2) <math>\begin{cases} x = 4 \\ y = 3 \end{cases}</math> <math>\begin{cases} x = 3 \\ y = 4 \end{cases}</math> <math>\begin{cases} x = -4 \\ y = -3 \end{cases}</math> <math>\begin{cases} x = -3 \\ y = -4 \end{cases}</math></p>	<p>(3) <math>\begin{cases} x = \pm 2 \\ y = \pm 2 \end{cases}</math> <math>\begin{cases} x = \pm 2 \\ y = \mp 1 \end{cases}</math></p> <p>(4) <math>\begin{cases} x = 3 \\ y = 2 \end{cases}</math> <math>\begin{cases} x = 2 \\ y = 3 \end{cases}</math></p>

151	152
<p>(1) <math>x + 3</math>                      (3) <math>2x + 5y</math></p> <p>(2) <math>x + 3</math>                      (4) <math>2x - 3y</math></p>	<p>(1) <math>x^2 + 2x + 4</math></p> <p>(2) <math>x^2 - 2x + 4</math></p>
151	152
<p>(5) <math>4x^2 + 2x - 3</math></p> <p>(6) <math>x^2 + 2x + 3</math></p>	<p>(3) <math>a^2 - ab + b^2</math></p> <p>(4) <math>x^3 + x^2y + xy^2 + y^3</math></p>

J 153–156

153	154
<p>(1) Quotient <math>2x + 6</math> Remainder <math>7x + 8</math></p>	<p>1. (1) Quotient <math>x^2 + 6x + 7</math> Remainder 17</p> <p>(2) <math>(x - 2)(x^2 + 6x + 7) + 17</math></p>
153	154
<p>(2) Quotient <math>2a^2 - a - 4</math> Remainder 6</p> <p>(3) Quotient <math>x - 2y - 1</math> Remainder <math>y - 4</math></p>	<p>2. (1) ( <math>2x^2 - x - 4</math> ) <span style="border: 1px solid black; padding: 0 5px;">6</span></p> <p>(2) <math>B(x^2 - x + 2) + 34</math></p>

155	156
<p>(1) <math>B(x^3 + 4x^2 + 4x + 3) + 18</math></p> <p>(2) <math>B(x^3 + 12x^2 - 6x + 5) + 5</math></p>	<p>1. <math>A = x^3 + x^2 + 2x + 5</math></p> <p>2. <math>A = 2x^3 - x^2 - 4x + 1</math></p>
155	156
<p>(3) <math>B(x^2 + 3x - 14) + 51x - 19</math></p> <p>(4) <math>B(x + 3) + 3</math></p>	<p>3. <math>A = x^4 + 2x^2 + x + 2</math></p> <p>4. <math>A = x^4 + x^3 + 2x^2 + 3x + 1</math></p>

J 157–160

157	158
<div>1.</div> <div><math>B = x^2 - x + 1</math></div>	<div>1.</div> <div>(1) <math>2 + \frac{3}{x - 2}</math></div> <div>(2) <math>-2 + \frac{7}{x + 3}</math></div> <div>(3) <math>2 + \frac{7}{2x - 1}</math></div>
157	158
<div>2.</div> <div><math>a = 3, \ b = 5</math></div>	<div>2.</div> <div>(1) <math>x + 1 + \frac{6}{x - 2}</math></div> <div>(2) <math>2x - 3 + \frac{4}{x + 1}</math></div> <div>(3) <math>x - 2 + \frac{4}{2x + 1}</math></div>



159	160
<p>(1) <math>\frac{-2x+20}{(x-4)(x-7)}</math></p> <p><math>\left[ = -\frac{2(x-10)}{(x-4)(x-7)} \right]</math></p>	<p>1.</p> <p>Quotient <math>x^2 + 4x + 2</math></p> <p>Remainder <math>28x + 8</math></p> <p>2.</p> <p><math>a = 2, b = 2</math></p>
159	160
<p>(2) <math>\frac{3a^2+1}{a(a-1)(a+1)}</math></p> <p>(3) <math>\frac{6x+18}{(x+1)(x+2)(x+4)(x+5)}</math></p> <p><math>\left[ = \frac{6(x+3)}{(x+1)(x+2)(x+4)(x+5)} \right]</math></p> <div data-bbox="112 1404 554 1476" style="border: 1px solid black; border-radius: 10px; padding: 5px; margin-top: 20px;"> <p>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</p> </div>	<p>3.</p> <p>Quotient <math>x^2 + 3x + 1</math></p> <p>Remainder <math>-10x + 1</math></p>

J 161–164

161	162
<div>1.</div> <div><math>a = 8</math></div>	<div>(1) <div>3</div> <div>3</div> <div>3</div> 27</div> <div>Quotient <math>x^2 + 4x + 7</math></div> <div>Remainder 27</div> <div><div><math>P( 3 )</math></div></div>
161	162
<div>2.</div> <div><math>a = 5</math></div>	<div>(2) <math>P( -2 ) = 12</math></div> <div>Quotient <math>x^2 - x - 3</math></div> <div>Remainder 12</div> <div><div><math>x + 2</math></div></div>
<div>3.</div> <div><math>a = 5</math></div>	
<div>4.</div> <div><math>a = 5</math></div>	<div>(3) <math>P( -1 ) = 11</math></div> <div>Quotient <math>x^2 - 5</math></div> <div>Remainder 11</div> <div><div><math>x + 1</math></div> <div><math>P( -1 )</math></div></div>

163	164
<p>1.</p> <p>(1) <math>-15</math></p> <p>(2) <math>-50</math></p> <p>(3) <math>-a^3 - a^2 + a + 2</math></p> <p>(4) <math>-\frac{23}{16}</math></p> <p>(5) <math>-52</math></p>	<p>1.</p> <p><math>a = 5</math></p> <p>2.</p> <p><math>a = 5</math></p> <p>3.</p> <p><math>a = \frac{1}{4}, \frac{1}{2}</math></p>
163	164
<p>2.</p> <p>(1) <math>3</math></p> <p>(2) <math>18</math></p> <p>(3) <math>\frac{32}{9}</math></p> <p>(4) <math>2a^2 + 3a + 4</math></p> <p>(5) <math>2a^2 + a + 3</math></p>	<p>4.</p> <p><math>a = -4, b = 1</math></p>

J 165–168

165	166
<div>1. <math>P(x) = (x - 2)(x + 3)Q(x) + ax + b</math></div> <div>2. <math>P(x) = (x + 1)(x + 4)Q(x) + ax + b</math></div> <div>3. <math>P(x) = (x^2 - x - 2)Q(x) + ax + b</math> <math>\quad [= (x - 2)(x + 1)Q(x) + ax + b]</math></div>	<div>1. <math>2x - 1</math></div>
165	166
<div>4. <math>P(x) = (x - 1)(x - 2)(x + 3)Q(x) + ax^2 + bx + c</math></div> <div>5. <math>P(x) = (x + 1)(x + 2)(x + 3)Q(x) + ax^2 + bx + c</math></div> <div>6. <math>P(x) = (x^3 - x - 1)Q(x) + ax^2 + bx + c</math></div> <div><div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div></div>	<div>2. <math>x</math></div> <div>3. <math>-2x + 8</math></div>

167	168
<p>1.</p> $x^2 + 2x + 3$	<p>1.</p> $-2$
167	168
<p>2.</p> $-2x^2 + 10x - 7$	<p>2.</p> $2x - 1$

J 169–172

169	170
	<div>1.</div> <div>(1) <math>a = -8</math></div> <div>(2) <math>a = -4</math></div> <div>(3) <math>a = 4</math></div> <div>(4) <math>a = 7</math></div>
169	170
<div>1.</div> <div><math>-1 + 2\sqrt{3} \, i</math></div>	<div>2.</div> <div><math>x^2 + 2</math></div>

171	172
<p>(1) <math>x = 2, \pm i</math></p> <p>(2) <math>x = 1, \frac{3 \pm \sqrt{7}i}{4}</math></p>	<p>(1) <math>P(x) = (x+1)^2(x-2)</math>  <math>[P(x) = (x-2)(x+1)^2]</math></p>
171	172
<p>(3) <math>x = -1, \frac{1 \pm \sqrt{3}i}{2}</math></p> <p>(4) <math>x = 1, \frac{-1 \pm \sqrt{3}i}{2}</math></p> <p>(5) <math>x = 2, -1 \pm \sqrt{3}i</math></p> <p>(6) <math>x = \pm i, \pm 1</math></p>	<p>(2) <math>P(x) = (x-1)(x^2+3x+3)</math></p> <p>(3) <math>P(x) = (x+1)(x+3)(x+2)</math></p> <p>(4) <math>P(x) = (x-1)(x+3)(x+1)</math></p> <div data-bbox="602 1404 1047 1481"> <p>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</p> </div>

J 173–176

173	174
<p>(1) <math>P(x) = (x - 5)(x^2 + x + 1)</math></p> <p>(2) <math>P(x) = (x - 6)(x^2 - x + 1)</math></p>	<p>(1) <math>P(x) = (2x - 1)(x^2 + x + 1)</math></p> <p>(2) <math>P(x) = (2x + 3)(x^2 - 2x + 3)</math></p>
173	174
<p>(3) <math>P(x) = (x + 2)(x^2 + 2x - 4)</math></p> <p>(4) <math>P(x) = (x + 3)(x^2 - 3x + 5)</math></p>	<p>(3) <math>P(x) = (2x - 1)(x^2 + 2x + 2)</math></p> <p>(4) <math>P(x) = (3x + 2)(x^2 - 2x - 1)</math></p>
<div>NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT</div>	



175	176
<p>(1) <math>x = 2, \frac{2}{3}, -2</math></p>	<p>(1) <math>x = 1, -\frac{2}{3}, -2</math></p> <p>(2) <math>x = 2, \frac{3}{2}, -3</math></p>
175	176
<p>(2) <math>x = \frac{1}{2}, -\frac{1}{3}, -\frac{1}{2}</math></p> <p>(3) <math>x = \frac{3}{2}, \frac{-1 \pm \sqrt{17}}{4}</math></p>	<p>(3) <math>x = 1, 2, -2 \pm \sqrt{2}</math></p>

J 177–180

177	178
<div>1.</div> <div>(1) <math>x = 3, -3 \pm \sqrt{11} i</math></div>	<div>1.</div> <div>(1) <math>a = -3, b = -6</math></div> <div>(2) <math>x = 1</math></div>
177	178
<div>2.</div> <div><math>a = -5, x = 2 \pm \sqrt{2}</math></div>	<div>2.</div> <div><math>a = -4, b = -12, x = \pm 2</math></div>

179	180
<p>1.</p> <p>(1) <math>\left(x + \frac{1}{x}\right)</math>  <math>\left(x + \frac{1}{x}\right)</math>  <math>\left(x + \frac{1}{x}\right)</math> <input type="text" value="5"/>  <input type="text" value="5"/></p> <p>(2) <input type="text" value="5"/>  <input type="text" value="5"/> <input type="text" value="1"/>  <input type="text" value="5"/>  <math>\frac{\quad}{2}</math> <input type="text" value="-1"/>  <input type="text" value="5"/>  <math>\frac{\quad}{2}</math> <input type="text" value="-1"/>  <math>x = \frac{1}{2}, 2, \frac{-1 \pm \sqrt{3}i}{2}</math></p>	<p>1.</p> <p>(1) <math>P(x) = (x+1)(x+3)(x+2)</math></p> <p>(2) <math>P(x) = (2x-1)(x^2+2x+2)</math></p>
179	180
<p>2.</p> <p><math>x = \frac{1}{2}, 2, -1</math></p>	<p>2.</p> <p>(1) <math>x = 2, \pm\sqrt{3}i</math></p> <p>(2) <math>x = -1, 2, 1 \pm \sqrt{2}i</math></p> <div data-bbox="602 1406 1046 1481" style="border: 1px solid black; border-radius: 10px; padding: 10px; text-align: center;">       NOTE: EQUIVALENT ANSWERS SHOULD BE MARKED CORRECT     </div>

J 181–184

181	182
(1) $a=2, b=\frac{1}{2}$	(1) $a=1, b=-1, c=2$
181	182
(2) $a=-2, b=3, c=1$	(2) $a=\frac{1}{2}, b=\frac{1}{2}, c=-1$  (3) $a=0, b=0, c=0$

183	184
<p>(1) <math>a = 3, b = 3, c = \frac{2}{3}</math></p> <p>or</p> <p><math>a = 3, b = 2, c = 1</math></p>	<p>1.</p> <p><math>a = -12, b = 16</math></p>
183	184
<p>(2) <math>a = -22, b = -11, c = -6</math></p> <p>(3) <math>a = \frac{3}{2}, b = \frac{1}{2}, c = -2</math></p>	<p>2.</p> <p><math>a = -6, b = 1</math></p>

J 185–187

185	186
$(1) \begin{cases} x = -1 \\ y = -2 \end{cases}$	1. $a = 2, \quad b = -1$
185	186
$(2) \begin{cases} x = 3 \\ y = 1 \end{cases} \quad \begin{cases} x = -1 \\ y = -1 \end{cases}$	2. $a = -1, \quad b = -2, \quad c = 3$

## 187

$$(1) \text{ LHS} = a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2$$

$$\begin{aligned} \text{RHS} &= a^2c^2 + 2abcd + b^2d^2 + a^2d^2 - 2abcd + b^2c^2 \\ &= a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2 \end{aligned}$$

Thus, LHS = RHS

## 187

$$(2) \text{ LHS} = x^2y^2 - x^2 - y^2 + 1$$

$$\begin{aligned} \text{RHS} &= (x^2y^2 + 2xy + 1) - (x^2 + 2xy + y^2) \\ &= x^2y^2 - x^2 - y^2 + 1 \end{aligned}$$

Thus, LHS = RHS

$$\begin{aligned} (3) \text{ LHS} &= a^2b + abc + a^2c + ab^2 + b^2c + abc + abc + bc^2 + ac^2 \\ &= a^2b + a^2c + ab^2 + b^2c + bc^2 + ac^2 + 3abc \end{aligned}$$

$$\begin{aligned} \text{RHS} &= (ab + ac + b^2 + bc)(c + a) + abc \\ &= abc + a^2b + ac^2 + a^2c + b^2c + ab^2 + bc^2 + abc + abc \\ &= a^2b + a^2c + ab^2 + b^2c + bc^2 + ac^2 + 3abc \end{aligned}$$

Thus, LHS = RHS

$$\left[ \begin{array}{l} \text{Alternative Solution} \\ \text{LHS} = [(a+b)+c][ab+c(a+b)] \\ \quad = ab(a+b) + c(a+b)^2 + abc + c^2(a+b) \\ \quad = (a+b)[ab+(a+b)c+c^2] + abc \\ \quad = (a+b)(a+c)(b+c) + abc \\ \quad = \text{RHS} \end{array} \right]$$

## 188

1.

If  $a + b = 1$ , then  $b = -a + 1 \dots \textcircled{1}$

Substituting  $\textcircled{1}$  into both sides,

$$\text{LHS} = a^2 + (-a + 1)^2 + 1 = 2a^2 - 2a + 2$$

$$\begin{aligned} \text{RHS} &= 2[a + (-a + 1) - a(-a + 1)] \\ &= 2(a^2 - a + 1) \\ &= 2a^2 - 2a + 2 \end{aligned}$$

Thus,  $\text{LHS} = \text{RHS}$

## 188

2.

$$\begin{aligned} (\text{LHS}) - (\text{RHS}) &= a^2 - bc - b^2 + ca \\ &= (a + b)(a - b) + c(a - b) \\ &= (a - b)(a + b + c) \\ &= 0 \end{aligned}$$

Thus,  $\text{LHS} = \text{RHS}$



## 189

1.

From  $a + b + c = 0$ ,

$$\begin{cases} a + b = -c \quad \dots \textcircled{1} \\ b + c = -a \quad \dots \textcircled{2} \\ c + a = -b \quad \dots \textcircled{3} \end{cases}$$

Substituting  $\textcircled{1}$ ,  $\textcircled{2}$  and  $\textcircled{3}$  into the LHS,

$$\begin{aligned} \text{LHS} &= \frac{1}{a}(-a) + \frac{1}{b}(-b) + \frac{1}{c}(-c) + 3 \\ &= -1 - 1 - 1 + 3 = 0 \end{aligned}$$

Thus, LHS = RHS

## 189

2.

If  $a + b + c = 0$ , then  $c = -(a + b)$ 

$$\begin{aligned} \text{LHS} &= a^3 + b^3 - (a + b)^3 \\ &= a^3 + b^3 - a^3 - 3a^2b - 3ab^2 - b^3 \\ &= -3a^2b - 3ab^2 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= -3ab(a + b) \\ &= -3a^2b - 3ab^2 \end{aligned}$$

Thus, LHS = RHS

## 190

1.

$$a = 1, b = 3, c = 3, d = 2$$

2.

$$\text{LHS} = \frac{y+1+x+1}{(x+1)(y+1)} = \frac{x+y+2}{xy+x+y+1} \dots \textcircled{1}$$

Substituting  $xy = 1$  into  $\textcircled{1}$ ,

$$\text{LHS} = \frac{x+y+2}{1+x+y+1} = \frac{x+y+2}{x+y+2} = 1$$

Thus, LHS = RHS

$$\left[ \begin{array}{l} \text{Alternative Solution} \\ (\text{LHS}) - (\text{RHS}) = \frac{1}{x+1} + \frac{1}{y+1} - 1 \\ = \frac{y+1+x+1-(x+1)(y+1)}{(x+1)(y+1)} \\ = \frac{y+1+x+1-xy-x-y-1}{(x+1)(y+1)} \\ = \frac{1-xy}{(x+1)(y+1)} = 0 \end{array} \right]$$

## 190

3.

$$\begin{aligned} \text{LHS} &= \frac{a}{b} + \frac{a}{c} + \frac{b}{c} + \frac{b}{a} + \frac{c}{a} + \frac{c}{b} + 3 \\ &= \frac{b+c}{a} + \frac{a+c}{b} + \frac{a+b}{c} + 3 \dots \textcircled{1} \end{aligned}$$

Since  $a+b+c=0$ ,

$$\begin{cases} b+c = -a \\ a+c = -b \\ a+b = -c \end{cases}$$

Substituting into  $\textcircled{1}$ ,

$$\begin{aligned} \text{LHS} &= \frac{-a}{a} + \frac{-b}{b} + \frac{-c}{c} + 3 \\ &= -1 - 1 - 1 + 3 = 0 \end{aligned}$$

Thus, LHS = RHS

## 191

$$(1) \frac{a+b}{a+2b} = \frac{c+d}{c+2d} \quad \dots(1)$$

$$\text{Let } \frac{a}{b} = \frac{c}{d} = k,$$

$$a = bk, \quad c = dk \quad \dots(2)$$

Substituting (2) into (1),

$$\text{LHS} = \frac{bk+b}{bk+2b} = \frac{b(k+1)}{b(k+2)} = \frac{k+1}{k+2}$$

$$\text{RHS} = \frac{dk+d}{dk+2d} = \frac{d(k+1)}{d(k+2)} = \frac{k+1}{k+2}$$

Thus, LHS = RHS

## 191

$$(2) \frac{a(a+c)}{b(b+d)} = \frac{c^2}{d^2} \quad \dots(1)$$

$$\text{Let } \frac{a}{b} = \frac{c}{d} = k,$$

$$a = bk, \quad c = dk \quad \dots(2)$$

Substituting (2) into (1),

$$\text{LHS} = \frac{bk(bk+dk)}{b(b+d)} = \frac{bk^2(b+d)}{b(b+d)} = k^2$$

$$\text{RHS} = \frac{(dk)^2}{d^2} = \frac{d^2k^2}{d^2} = k^2$$

Thus, LHS = RHS

$$(3) \frac{b^2+d^2}{a^2+c^2} = \frac{b^2-d^2}{a^2-c^2} \quad (b^2 \neq d^2) \quad \dots(1)$$

$$\text{Let } \frac{a}{b} = \frac{c}{d} = k,$$

$$a = bk, \quad c = dk \quad \dots(2)$$

Substituting (2) into (1),

$$\text{LHS} = \frac{b^2+d^2}{b^2k^2+d^2k^2} = \frac{b^2+d^2}{k^2(b^2+d^2)} = \frac{1}{k^2}$$

$$\text{RHS} = \frac{b^2-d^2}{b^2k^2-d^2k^2} = \frac{b^2-d^2}{k^2(b^2-d^2)} = \frac{1}{k^2}$$

Thus, LHS = RHS

J 192–194

192	193
1. $\frac{47}{50}$	1. $\frac{1}{4}$
192	193
2. $\frac{2}{9}$	2. (1) 2  (2) −1

## 194

1.

$$\begin{aligned}(\text{LHS}) - (\text{RHS}) &= \frac{x+y}{2} - \frac{2x+3y}{5} \\ &= \frac{x-y}{10}\end{aligned}$$

As  $x > y$ ,

$$x - y > 0$$

Therefore,  $(\text{LHS}) - (\text{RHS}) > 0$ Thus,  $\frac{x+y}{2} > \frac{2x+3y}{5}$  is true.

## 194

2.

$$\begin{aligned}(\text{LHS}) - (\text{RHS}) &= x^2 - 2x + 3 \\ &= (x-1)^2 + 2\end{aligned}$$

Since  $(x-1)^2 \geq 0$ ,

$$(x-1)^2 + 2 \geq 2 > 0$$

Therefore,  $x^2 > 2x - 3$

## 195

$$\begin{aligned}
 & 1. \\
 (1) \quad & (\text{LHS}) - (\text{RHS}) \\
 &= (a^2 + b^2)(x^2 + y^2) - (ax + by)^2 \\
 &= (a^2x^2 + a^2y^2 + b^2x^2 + b^2y^2) - (a^2x^2 + 2abxy + b^2y^2) \\
 &= a^2y^2 - 2abxy + b^2x^2 \\
 &= (ay - bx)^2 \geq 0
 \end{aligned}$$

$$\text{Thus, } (a^2 + b^2)(x^2 + y^2) \geq (ax + by)^2$$

LHS = RHS when  $ay - bx = 0$ , i.e. when  $ay = bx$

## 195

$$\begin{aligned}
 & 2. \\
 (1) \quad & (\text{LHS}) - (\text{RHS}) = a^2 + b^2 - 2(a - b - 1) \\
 &= a^2 - 2a + b^2 + 2b + 2 \\
 &= (a^2 - 2a + 1) + (b^2 + 2b + 1) \\
 &= (a - 1)^2 + (b + 1)^2
 \end{aligned}$$

Since  $(a - 1)^2 \geq 0$ , and  $(b + 1)^2 \geq 0$ ,

$$(a - 1)^2 + (b + 1)^2 \geq 0$$

Therefore,  $a^2 + b^2 \geq 2(a - b - 1)$

LHS = RHS when  $a - 1 = 0$  and  $b + 1 = 0$ , i.e. when

$$a = 1, b = -1.$$

## 196

1.

(1)  $a + b = 1$ , so  $b = 1 - a$ 

$$\begin{aligned}
 (\text{LHS}) - (\text{RHS}) &= a^2 + (1 - a)^2 - \frac{1}{2} \\
 &= 2a^2 - 2a + \frac{1}{2} \\
 &= 2\left(a - \frac{1}{2}\right)^2 \geq 0
 \end{aligned}$$

Therefore,  $a^2 + b^2 \geq \frac{1}{2}$ Thus, LHS = RHS when  $a = b = \frac{1}{2}$ 

## 196

2.

(1)  $a + b = 1$ , so  $b = 1 - a$ 

$$\begin{aligned}
 (\text{LHS}) - (\text{RHS}) &= a^2 + (1 - a)^2 - a(1 - a) \\
 &= 3a^2 - 3a + 1 \\
 &= 3\left(a - \frac{1}{2}\right)^2 + \frac{1}{4} \geq \frac{1}{4} > 0
 \end{aligned}$$

Thus,  $a^2 + b^2 > ab$ (2)  $a + b = 1$ , so  $b = 1 - a$ 

$$\begin{aligned}
 (\text{LHS}) - (\text{RHS}) &= a + (1 - a) - a(1 - a) \\
 &= a^2 - a + 1 \\
 &= \left(a - \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4} > 0
 \end{aligned}$$

Thus,  $a + b > ab$

197

1.

$$\begin{aligned}
 (1) \quad (\text{LHS}) - (\text{RHS}) &= \frac{\sqrt{ab}(a+b) - 2ab}{a+b} \\
 &= \frac{\sqrt{ab}(a+b - 2\sqrt{ab})}{a+b} \\
 &= \frac{\sqrt{ab}(\sqrt{a} - \sqrt{b})^2}{a+b} \geq 0
 \end{aligned}$$

Therefore,  $\sqrt{ab} \geq \frac{2ab}{a+b}$

LHS = RHS when  $a = b$

197

2.

$$\begin{aligned}
 \frac{\text{LHS}}{2} &= \frac{\frac{x}{y} + \frac{y}{x}}{2} \geq \sqrt{\frac{x}{y} \cdot \frac{y}{x}} \\
 \frac{\frac{x}{y} + \frac{y}{x}}{2} &\geq 1
 \end{aligned}$$

Thus,  $\frac{x}{y} + \frac{y}{x} \geq 2$

LHS = RHS when  $x = y$



## 198

1.

$$(1) \frac{\text{LHS}}{2} = \frac{a + \frac{9}{a}}{2} \geq \sqrt{a \cdot \frac{9}{a}}$$

$$\frac{a + \frac{9}{a}}{2} \geq 3$$

$$\text{Therefore, } a + \frac{9}{a} \geq 6$$

$$\text{LHS} = \text{RHS when } a = 3$$

$$(2) \frac{\text{LHS}}{2} = \frac{2a + \frac{1}{8a}}{2} \geq \sqrt{2a \cdot \frac{1}{8a}}$$

$$\frac{2a + \frac{1}{8a}}{2} \geq \frac{1}{2}$$

$$\text{Therefore, } 2a + \frac{1}{8a} \geq 1$$

$$\text{LHS} = \text{RHS when } a = \frac{1}{4}$$

## 198

2.

$$(1) \text{LHS} = \left(a + \frac{2}{b}\right)\left(b + \frac{8}{a}\right) = 10 + ab + \frac{16}{ab}$$

$$a > 0 \text{ and } b > 0, \text{ so } ab > 0 \text{ and } \frac{16}{ab} > 0$$

Therefore,

$$\begin{aligned} 10 + ab + \frac{16}{ab} &\geq 10 + 2\sqrt{ab \cdot \frac{16}{ab}} \\ &= 10 + 8 \\ &= 18 \end{aligned}$$

$$\text{Thus, } \left(a + \frac{2}{b}\right)\left(b + \frac{8}{a}\right) \geq 18$$

$$\text{LHS} = \text{RHS when } ab = 4$$

1.

$$\text{LHS} = (a+b)\left(\frac{1}{a} + \frac{1}{b}\right) = 2 + \frac{a}{b} + \frac{b}{a}$$

$$a > 0 \text{ and } b > 0, \text{ so } \frac{a}{b} > 0 \text{ and } \frac{b}{a} > 0$$

Therefore,

$$2 + \frac{a}{b} + \frac{b}{a} \geq 2 + 2\sqrt{\frac{a}{b} \cdot \frac{b}{a}} = 4$$

$$\text{Thus, } (a+b)\left(\frac{1}{a} + \frac{1}{b}\right) \geq 4$$

$$\text{LHS} = \text{RHS when } a = b$$

2.

$$\text{LHS} = \left(\frac{b}{a} + \frac{d}{c}\right)\left(\frac{a}{b} + \frac{c}{d}\right) = 2 + \frac{bc}{ad} + \frac{ad}{bc}$$

$$a > 0, b > 0, c > 0, d > 0, \text{ so } \frac{bc}{ad} > 0 \text{ and } \frac{ad}{bc} > 0$$

Therefore,

$$2 + \frac{bc}{ad} + \frac{ad}{bc} \geq 2 + 2\sqrt{\frac{bc}{ad} \cdot \frac{ad}{bc}} = 4$$

$$\text{Thus, } \left(\frac{b}{a} + \frac{d}{c}\right)\left(\frac{a}{b} + \frac{c}{d}\right) \geq 4$$

$$\text{LHS} = \text{RHS when } ad = bc$$

## 200

1.

$$\frac{11}{14}$$

2.

$$(a+b)\left(\frac{1}{a} + \frac{4}{b}\right) = 5 + \frac{4a}{b} + \frac{b}{a}$$

$$a > 0, b > 0, \text{ so } \frac{4a}{b} > 0 \text{ and } \frac{b}{a} > 0$$

$$\begin{aligned} 5 + \frac{4a}{b} + \frac{b}{a} &\geq 5 + 2\sqrt{\frac{4a}{b} \cdot \frac{b}{a}} \\ &= 5 + 4 \\ &= 9 \end{aligned}$$

$$\text{Therefore, } (a+b)\left(\frac{1}{a} + \frac{4}{b}\right) \geq 9$$

$$\text{LHS} = \text{RHS when } 2a = b$$

## 200

3.

$$\begin{aligned} (\text{LHS}) - (\text{RHS}) &= a^2 + b^2 + c^2 - (ab + bc + ca) \\ &= a^2 - a(b+c) + (b^2 - bc + c^2) \\ &= \left(a - \frac{b+c}{2}\right)^2 - \frac{(b+c)^2}{4} + b^2 - bc + c^2 \\ &= \left(a - \frac{b+c}{2}\right)^2 + \frac{3b^2 - 6bc + 3c^2}{4} \\ &= \left(a - \frac{b+c}{2}\right)^2 + \frac{3}{4}(b^2 - 2bc + c^2) \\ &= \left(a - \frac{b+c}{2}\right)^2 + \frac{3}{4}(b-c)^2 \geq 0 \end{aligned}$$

$$\text{Thus, } a^2 + b^2 + c^2 \geq ab + bc + ca$$

$$\text{LHS} = \text{RHS when } a - \frac{b+c}{2} = 0 \text{ and } b-c = 0, \text{ i.e. when } a = b = c.$$